Effective Field Theory Beyond the Standard Model

Scott Willenbrock[1,2] and Cen Zhang[3]

[1]Department of Physics, University of Illinois at Urbana–Champaign, Urbana, Illinois 61801

[2]Fermi National Accelerator Laboratory, Batavia, Illinois 60510

[3]Centre for Cosmology, Particle Physics and Phenomenology (CP3), Université Catholique de Louvain, B-1348 Louvain-la-Neuve, Belgium; email: cen.zhang@uclouvain.be

Abstract

We review the effective field theory approach to physics beyond the Standard Model using dimension-six operators. Topics include the choice of operator basis, electroweak boson pair production, precision electroweak physics (including one-loop contributions), and Higgs physics. By measuring the coefficients of dimension-six operators with good accuracy, we can hope to infer some or all of the features of the theory that lies beyond the Standard Model.

Contents

1. INTRODUCTION

There are two methods to search for physics beyond the Standard Model of particle physics. One is to search for new particles. The other is to search for new interactions of known particles. The latter is the subject of this review.

There exists a long history of physicists supplementing established theories with new interactions, followed by experiments to measure or bound these interactions. In this review we concentrate on particle physics at the energy frontier, in which the established theory is the Standard Model, including the Higgs particle that was discovered in July 2012 by the ATLAS (1) and CMS (2) Collaborations at the CERN Large Hadron Collider (LHC) from proton–proton collisions at 7 and 8 TeV. This particle has been studied in some detail, and it behaves as the Standard Model Higgs particle to a good approximation, so we treat it thus. As of fall 2013, no particles beyond the Standard Model have been found; we await proton–proton collisions at 13–14 TeV.

The first question that arises is how one should go about parameterizing the hypothesized new interactions. However one extends the Standard Model, it should not do violence with the framework of the Standard Model itself. Because the Standard Model is a quantum field theory, the extension should also be a quantum field theory and respect all the axioms of quantum field theory.

The new interactions should respect the $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge symmetries of the Standard Model. Whereas the electroweak interaction is spontaneously broken at energies below the mass of the Higgs boson, $m_H \approx 126$ GeV, it is unbroken above this energy. Thus, the gauge symmetries of the Standard Model are unbroken at the energy frontier.

We assume that the new interactions decouple from the Standard Model in the limit that the energy scale characterizing the new interactions goes to infinity. This is not true of all theories beyond the Standard Model, but it is a good working assumption and is consistent with what we know at present.

Many sectors of the Standard Model have been tested with an accuracy that goes beyond leading order in perturbation theory. We therefore desire that any new interactions that are introduced allow for unambiguous calculations beyond leading order. Ideally, one should be able to calculate any process at any order in both the Standard Model interactions and the new interactions.

All of the above properties are satisfied by an effective field theory (3–5). As the name implies, it is a quantum field theory, and it is straightforward to implement the gauge symmetries of the Standard Model. It carries with it an energy scale, Λ, that we interpret as the scale of new physics and that we assume to be greater than the Higgs mass. New interactions are constructed from

Standard Model fields and have a coefficient proportional to an inverse power of Λ; thus, the Standard Model is recovered in the limit $\Lambda \to \infty$. The new interactions are compatible with the calculation of Standard Model radiative corrections. The new interactions may be calculated to any desired order in $1/\Lambda$, with the caveat that the introduction of additional interactions may be necessary at each order in $1/\Lambda$.

A benefit of thinking of the Standard Model as an effective field theory is that this explains some of its mysteries. Why does the Standard Model contain only renormalizable interactions, and not the more complicated nonrenormalizable interactions? The answer is that the nonrenormalizable interactions have coefficients that are proportional to inverse powers of the scale of new physics, Λ, and are therefore suppressed at energies less than Λ. If one writes down every interaction constructed from Standard Model fields with coefficients that are dimensionless or of positive-mass dimension, one arrives at the Standard Model.

Another benefit of thinking of the Standard Model as an effective field theory is that it gives some guidance as to what new interactions to look for. One would expect the dominant new interactions to be the ones that are least suppressed by inverse powers of Λ. In the absence of any restrictions, the new interactions that are the least suppressed are proportional to the first power of $1/\Lambda$. There is only one such interaction (for one generation of fermions), and it gives rise to Majorana masses for neutrinos (6). Because our focus is on the energy frontier, we do not discuss this interaction any further.

The interactions that are suppressed by $1/\Lambda^2$ come in two classes. The first consists of the interactions that violate baryon number (B) and lepton number (L). There are five such interactions (for one generation) (6–8), and the ones involving the light quarks and leptons are highly constrained by experiments such as proton decay. The second class consists of the interactions that respect B and L, which are less constrained. There are a staggering number of these interactions: The number of independent interactions of this type is 59 (for one generation) (9). By the same counting, the number of interactions in the Standard Model is only 14 (for one generation). Thus, we gain a renewed appreciation for the simplicity of the Standard Model.

The interaction that gives rise to Majorana masses for neutrinos violates L by two units. In fact, all interactions that are proportional to an odd power of $1/\Lambda$ violate B and/or L (10). We restrict our attention to the interactions that conserve B and L, given that these are the least constrained and therefore are the most likely to be relevant to experiments at the energy frontier.

The effective field theory of the Standard Model that conserves B and L is thus

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \sum_i \frac{c_i}{\Lambda^2} \mathcal{O}_i + \cdots, \qquad \qquad 1.$$

where c_i is a dimensionless coefficient[1] and $\mathcal{O}_i$ is an operator[2] constructed from Standard Model fields. Because a Lagrangian is of mass dimension four, the operators $\mathcal{O}_i$ are of mass dimension six. The ellipsis refers to operators of dimension eight, ten, and so on. The expectation is that the leading effects of new physics will be represented by the dimension-six operators because they are the least suppressed.

An example of an effective field theory is given by a new Z' boson that couples to Standard Model fermions. At energies below the Z' mass, one does not observe the new particle directly but, rather, a new four-fermion interaction of Standard Model fermions. Because fermion fields are of mass dimension 3/2, this interaction is represented by a dimension-six operator. The Z'

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[1]These are often referred to as Wilson coefficients to honor Ken Wilson's pioneering work on effective field theory.

[2]If $\mathcal{O}_i$ is not hermitian, its hermitian conjugate should be added to Equation 1.

propagator contributes a factor of $1/M_{Z'}^2$ to the interaction, so the scale of new physics, Λ, is the mass of the Z' boson. The dimensionless coefficient of the dimension-six operator, c_i, is the product of the Z' couplings to the Standard Model fermions. The reader may recognize that the above example is analogous to the Fermi theory of the weak interaction, which is the effective field theory of the weak interaction at energies below the W mass.

This example illustrates that an effective field theory is not intended to be valid to arbitrarily high energies, but only up to the scale of new physics, Λ. Beyond that energy, the new particles must be included explicitly. One may then construct a new effective field theory from the Standard Model particles and the new particles that have been discovered.

At this time there is no established evidence for the presence of any dimension-six operators. One may instead use the world's data to place bounds on the coefficients of these operators. In this way we quantify the accuracy with which the new interactions are excluded. The advantage of this approach is that it does not make reference to any particular experiment. Different experiments place bounds on different combinations of dimension-six operators. If every analysis uses the same approach, we can combine the world's data in a consistent way.

If we were fortunate enough to find that the coefficients of some of the dimension-six operators were nonzero, it would amount to the discovery of new physics. By measuring the coefficients with good accuracy, one can hope to infer some or all of the features of the theory that underlie the effective field theory. There is a historical precedent in the theory of the weak interaction. Fermi's original theory of the weak interaction, developed shortly after the discovery of the neutron in 1932, was based on vector currents; it took many years of experimentation to establish that the true theory involved vector minus axial-vector ($V-A$) currents (11, 12), ultimately leading to the introduction of the $SU(2)_L$ gauge group as a fundamental aspect of the electroweak theory (13–15).

The coefficients of dimension-six operators are dimensionful: c_i/Λ^2. Unfortunately, measuring these coefficients does not reveal the scale of new physics, Λ, only the ratio c_i/Λ^2. For example, the $V-A$ effective theory of the weak interaction by itself did not reveal the mass of the W boson, only the ratio $G_F/\sqrt{2} = g^2/8m_W^2$. Searches were performed for W bosons as light as a few GeV (16). However, once the $SU(2)_L \times U(1)_Y$ theory was proposed and the weak neutral currents were measured, there was enough information to infer the mass of the W (and Z) boson because it was possible to derive the weak coupling g from the electromagnetic coupling e via $e = g \sin\theta_W$, where $\sin\theta_W$ is extracted from weak neutral-current experiments. Thus, once one has an underlying theory in mind, it may be possible to infer the scale of new physics, Λ.

2. OPERATOR BASIS

If one writes down all dimension-six operators that can be constructed from Standard Model fields and that respect the Standard Model gauge symmetries (as well as B and L), one arrives at a list that is longer than the 59 operators mentioned above. However, many of these operators are redundant; they are equivalent to a linear combination of other operators. These linear relations correspond to the Standard Model equations of motion or other identities (17). This redundancy is a feature of dimension-six operators that is unfamiliar from the Standard Model, in which one does not encounter redundant operators.

Because of this redundancy, there exists a great deal of flexibility in which set of 59 operators to use. Any set of 59 independent operators constitutes a good basis. However, there is no physically preferred basis: Any basis can be used to describe the data. A particular experimental measurement generally depends on only a few of the 59 operators in a given basis.

One can place a bound on the coefficient of a particular operator by assuming that all the other operators in that basis have vanishing coefficients. However, this is an ad hoc assumption,

unless one has in mind an underlying theory that produces this particular pattern of coefficients. In the absence of an underlying theory, one should include every dimension-six operator that contributes to the calculation of a physical process. Thus, each experimental measurement will generally bound a set of dimension-six operators.

Several operator bases are currently popular, and we characterize them as follows.

1. The first attempt to construct a complete basis goes back to Reference 18, which contains 80 dimension-six operators (for one generation). Gradually, it was discovered that some of these operators are redundant (19–21), which eventually led to the basis of 59 operators (9). Thus, the basis used in Reference 9 may be viewed as the descendant of the first attempt at constructing a complete basis (18) and is, thus, historically significant, at the very least. Below, we refer to this basis as the BW basis, with apologies to those who contributed to its development.

2. A basis that maximizes the number of operators involving only Higgs and electroweak gauge bosons was constructed in Reference 22. Given that this paper did not discuss operators involving fermions, it was not intended to be a complete basis. Nevertheless, it is widely used for studies of Higgs and weak boson physics, and it can be extended to a complete basis by adding operators involving fermions. We refer to this basis as the HISZ basis. When it is extended to include operators involving fermions, one may choose to eliminate some of the HISZ operators via equations of motion.

3. A third class of bases began with Reference 23 and has been developed into a complete basis (24–28), motivated in part by the discovery of the Higgs boson. We refer to this class of bases as the GGPR basis, again with apologies to those who contributed to its development.[3]

Other bases are also in use, but we do not attempt to summarize them. For example, the basis used in Reference 29 is a cross between the BW basis and the GGPR basis.

Table 1 shows the *CP*-even operators containing only electroweak boson fields in these three bases. If the operators look strange and unfamiliar, it is because they are: After all, they are not present in the Standard Model, so we are not accustomed to them. It takes some time to get used to them, and it does not help that three different bases are presently in use, with three different notations and normalizations. For example, $\mathcal{O}_W$ represents three completely different operators in the three bases. We encounter some of these operators below.

Numerous dimension-six operators contain fermions, and we do not list them here. They tend to be nearly the same in the three popular bases, although there are differences that can be important. For example, the basis used in Reference 28, which is in the GGPR class of bases, uses equations of motion to eliminate two of the operators containing fermions present in the BW basis.

Some operators are *CP* odd. At tree level, they interfere with the Standard Model only if the observable is constructed using triple-product correlation of momenta and/or spins; thus, the effect can be observed only in processes in which there are at least four independent momenta and/or spins that can be measured. We do not consider *CP*-odd operators in this review.

If the underlying theory is a weakly coupled renormalizable gauge theory, it is possible to classify dimension-six operators as being potentially generated at tree level or at one loop (31). This classification is cleanest in a basis containing the maximum number of potentially tree-generated operators (32). Both the BW basis and the GGPR basis satisfy this criterion, whereas the HISZ basis does not. It has been argued that this classification also applies to strongly coupled

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[3]This basis is sometimes referred to by the acronym SILH (strongly interacting light Higgs), which is the title of the GGPR paper (23). We prefer to avoid this label because this basis is generally applicable.

Table 1 The *CP*-even operators containing only electroweak boson fields[a]

BW[b]	HISZ[c]	GGPR[d]
$\mathcal{O}_W = \epsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	$\mathcal{O}_{WWW} = \mathrm{Tr}[\hat{W}_{\mu\nu} \hat{W}^{\nu\rho} \hat{W}_\rho^\mu]$	$\mathcal{O}_{3W} = \frac{1}{3!} g\epsilon_{abc} W_\mu^{a\nu} W_{\nu\rho}^b W^{c\rho\mu}$
$\mathcal{O}_{\varphi W} = \varphi^\dagger \varphi W_{\mu\nu}^I W^{I\mu\nu}$	$\mathcal{O}_{WW} = \Phi^\dagger \hat{W}_{\mu\nu} \hat{W}^{\mu\nu} \Phi$	—
$\mathcal{O}_{\varphi B} = \varphi^\dagger \varphi B_{\mu\nu} B^{\mu\nu}$	$\mathcal{O}_{BB} = \Phi^\dagger \hat{B}_{\mu\nu} \hat{B}^{\mu\nu} \Phi$	$\mathcal{O}_{BB} = g'^2 |H|^2 B_{\mu\nu} B^{\mu\nu}$
$\mathcal{O}_{\varphi WB} = \varphi^\dagger \sigma^I \varphi W_{\mu\nu}^I B^{\mu\nu}$	$\mathcal{O}_{BW} = \Phi^\dagger \hat{B}_{\mu\nu} \hat{W}^{\mu\nu} \Phi$	—
—	$\mathcal{O}_W = (D_\mu \Phi)^\dagger \hat{W}^{\mu\nu} (D_\nu \Phi)$	$\mathcal{O}_{HW} = ig(D^\mu H)^\dagger \sigma^a (D^\nu H) W_{\mu\nu}^a$
—	$\mathcal{O}_B = (D_\mu \Phi)^\dagger \hat{B}^{\mu\nu} (D_\nu \Phi)$	$\mathcal{O}_{HB} = ig'(D^\mu H)^\dagger (D^\nu H) B_{\mu\nu}$
—	$\mathcal{O}_{DW} = \mathrm{Tr}\left([D_\mu, \hat{W}_{\nu\rho}][D^\mu, \hat{W}^{\nu\rho}]\right)$	$\mathcal{O}_{2W} = -\frac{1}{2}\left(D^\mu W_{\mu\nu}^a\right)^2$
—	$\mathcal{O}_{DB} = -\frac{g'^2}{2}(\partial_\mu B_{\nu\rho})(\partial^\mu B^{\nu\rho})$	$\mathcal{O}_{2B} = -\frac{1}{2}\left(\partial^\mu B_{\mu\nu}\right)^2$
—	—	$\mathcal{O}_W = \frac{ig}{2}(H^\dagger \sigma^a \overleftrightarrow{D}^\mu H) D^\nu W_{\mu\nu}^a$
—	—	$\mathcal{O}_B = \frac{ig'}{2}(H^\dagger \overleftrightarrow{D}^\mu H) \partial^\nu B_{\mu\nu}$
$\mathcal{O}_{\varphi D} = (\varphi^\dagger D^\mu \varphi)^* (\varphi^\dagger D_\mu \varphi)$	$\mathcal{O}_{\Phi,1} = (D_\mu \Phi)^\dagger \Phi \Phi^\dagger (D^\mu \Phi)$	$\mathcal{O}_T = \frac{1}{2}(H^\dagger \overleftrightarrow{D}_\mu H)^2$
$\mathcal{O}_{\varphi\square} = (\varphi^\dagger \varphi)\square(\varphi^\dagger \varphi)$	$\mathcal{O}_{\Phi,2} = \frac{1}{2}\partial_\mu(\Phi^\dagger \Phi)\partial^\mu(\Phi^\dagger \Phi)$	$\mathcal{O}_H = \frac{1}{2}(\partial^\mu |H|^2)^2$
$\mathcal{O}_\varphi = (\varphi^\dagger \varphi)^3$	$\mathcal{O}_{\Phi,3} = \frac{1}{3}(\Phi^\dagger \Phi)^3$	$\mathcal{O}_6 = \lambda |H|^6$
—	$\mathcal{O}_{\Phi,4} = (D_\mu \Phi)^\dagger (D^\mu \Phi)(\Phi^\dagger \Phi)$	—

[a]Operators of three popular bases of dimension-six operators are listed. The operators in each row are either identical (up to normalization) or similar.
[b]The notation and normalization of the BW basis (18) are from Reference 9.
[c]The notation and normalization of the HISZ basis (22) are from References 22 and 30.
[d]The notation and normalization of the GGPR basis (23) are from Reference 27.

underlying theories that are minimally coupled (33), although the principle of minimal coupling (which is not a principle of the Standard Model) has been criticized (34).

Our own view is that one should fit the data with dimension-six operators without regard to their classification. If the data indicate that some dimension-six operators have nonvanishing coefficients, then the classification of the operators may help in unraveling the underlying theory.

3. WEAK BOSON PAIR PRODUCTION

As an example of the application of effective field theory to a physical process, let us consider weak boson pair production, specifically $W^+ W^-$ production. This process has been measured at LEP-II via $e^+ e^- \to W^+ W^-$ and at the Tevatron and LHC via $q\bar{q} \to W^+ W^-$.

Dimension-six operators affect this process in two different ways. First, they can affect the processes from which the input parameters α, G_F, and M_Z are derived. Second, dimension-six operators can contribute directly to this process. A given operator may even contribute in both ways.

Let us consider the dimension-six operator

$$\mathcal{O}_{WWW} = \mathrm{Tr}[\hat{W}_{\mu\nu} \hat{W}^{\nu\rho} \hat{W}_\rho^\mu], \qquad 2.$$

where

$$\hat{W}_{\mu\nu} = \frac{i}{2} g\sigma^a (\partial_\mu W_\nu^a - \partial_\nu W_\mu^a - g\epsilon^{abc} W_\mu^b W_\nu^c) \qquad 3.$$

is the $SU(2)_L$ field strength tensor. This operator is present in all three bases listed in **Table 1**, although with different notations and normalizations; we adopt the HISZ conventions for this discussion. This operator gives rise to three-, four-, five-, and six-point weak-boson
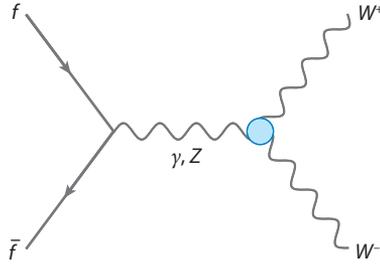
Figure 1

Feynman diagram for $f\bar{f} \to W^+ W^-$. The blue dot represents an interaction arising from a dimension-six operator.

interactions. For the process under consideration, it is the three-point interactions that matter, namely $\gamma W^+ W^-$ and $Z W^+ W^-$. These interactions give rise to the Feynman diagrams shown in **Figure 1**, which modify the differential cross section for $f\bar{f} \to W^+ W^-$.

Our only information about this operator at present (at tree level) is from $f\bar{f} \to W^+ W^-$. We can think of this operator as a modification of the triple gauge boson interactions $\gamma W^+ W^-$ and $Z W^+ W^-$ and can relate it to a standard parameterization of C- and P-conserving anomalous triple gauge boson couplings (35, 36),

$$\mathcal{L} = i g_{WWV} \left[g_1^V \left(W_{\mu\nu}^+ W^{-\mu} - W^{+\mu} W_{\mu\nu}^- \right) V^\nu + \kappa_V W_\mu^+ W_\nu^- V^{\mu\nu} + \frac{\lambda_V}{m_W^2} W_\mu^{+\nu} W_\nu^{-\rho} V_\rho^\mu \right],$$

where $V = \gamma, Z$; $W_{\mu\nu}^\pm = \partial_\mu W_\nu^\pm - \partial_\nu W_\mu^\pm$; $V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$; and the overall coupling constants are defined as $g_{WW\gamma} = e$ and $g_{WWZ} = e \cot \theta_W$. Electromagnetic gauge invariance implies that $g_1^\gamma = 1$, so there are five independent C- and P-conserving parameters: $g_1^Z, \kappa_\gamma, \kappa_Z, \lambda_\gamma$, and λ_Z. The relation of $\mathcal{O}_{WWW}$ to this Lagrangian is

$$\lambda_\gamma = \lambda_Z = c_{WWW} \frac{3 g^2 m_W^2}{2 \Lambda^2}, \qquad 4.$$

where c_{WWW}/Λ^2 is the coefficient of the operator $\mathcal{O}_{WWW}$ in the Lagrangian. The nontrivial result $\lambda_\gamma = \lambda_Z$ follows from restricting our attention to dimension-six operators. If we were to include dimension-eight operators, the equality of these two anomalous couplings would be violated (22). Dimension-eight operators are suppressed by $1/\Lambda^4$, so we expect these couplings to be equal to good accuracy.

It is well known that the presence of anomalous triple gauge boson couplings leads to a cross section for $f\bar{f} \to W^+ W^-$ that violates the unitarity bound at high energy. This finding led to the introduction of ad hoc energy-dependent form factors such that the cross section respects the unitarity bound at arbitrarily high energy (37). The effective field theory viewpoint makes it clear that this is an unnecessary complication (10). An effective field theory is valid only up to the scale of new physics, Λ, not to an arbitrarily high energy. The data must respect the unitarity bound, and because the effective field theory is intended only to describe the data, it will also respect the unitarity bound. It is not a concern that the theoretical cross section violates the unitarity bound beyond the energy where there are data (**Figure 2**).

The reason the dimension-six operator $\mathcal{O}_{WWW}$, via $\lambda_{\gamma,Z}$ (Equation 4), leads to a cross section that violates the unitarity bound at high energy, E, is that it introduces terms in the $f\bar{f} \to W^+ W^-$ amplitude that are proportional to $c_{WWW} E^2/\Lambda^2$. One way to place an upper bound on Λ is to calculate the energy at which the unitarity bound is violated, for a given value of c_{WWW}/Λ^2.
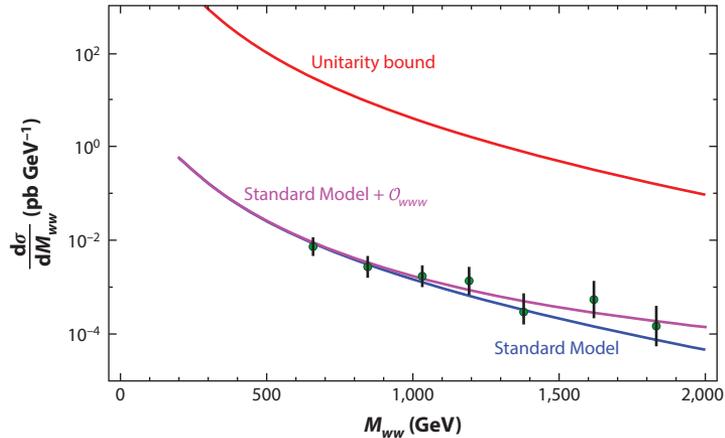
The invariant mass distribution for W^+W^- production at the LHC. The lowest (*blue*) curve is the Standard Model prediction; the middle (*purple*) curve includes the effect of the dimension-six operator $\mathcal{O}_{WWW}$; and the upper (*red*) curve is the unitarity bound. The data are hypothetical.

Doing so places an upper bound on the energy at which the effective field theory is valid and, hence, an upper bound on Λ. Such a calculation in the $V-A$ theory led to an upper bound on the W boson mass of ~ 700 GeV (38). The fact that the W boson mass is considerably lower than this upper bound reflects that the weak interaction is weakly coupled.

Unitarity bounds on anomalous triple gauge boson couplings should be viewed from the same perspective (39, 40). For a given value of an anomalous coupling, one can use the unitarity bound to calculate an upper bound on the scale of new physics, Λ. As the anomalous coupling tends to zero, the upper bound on the scale of new physics recedes to infinity.

In the HISZ basis, several other operators contribute directly to the anomalous gauge boson couplings. Restricting our attention to the CP-even operators, in addition to $\mathcal{O}_{WWW}$ there are

$$\mathcal{O}_W = (D_\mu \Phi)^\dagger \hat{W}^{\mu\nu} (D_\nu \Phi), \qquad\qquad 5.$$

$$\mathcal{O}_B = (D_\mu \Phi)^\dagger \hat{B}^{\mu\nu} (D_\nu \Phi), \qquad\qquad 6.$$

$$\mathcal{O}_{BW} = \Phi^\dagger \hat{B}_{\mu\nu} \hat{W}^{\mu\nu} \Phi, \qquad\qquad 7.$$

$$\mathcal{O}_{DW} = \mathrm{Tr}([D_\mu, \hat{W}_{\nu\rho}][D^\mu, \hat{W}^{\nu\rho}]). \qquad\qquad 8.$$

The last operator contributes to $\lambda_\gamma = \lambda_Z$,[4] whereas the first three operators contribute to the anomalous couplings $\Delta g_1^Z = g_1^Z - 1$, $\Delta\kappa_Z = \kappa_Z - 1$, and $\Delta\kappa_\lambda = \kappa_\lambda - 1$. However, one finds that there is a relation among these anomalous couplings,

$$\Delta g_1^Z = \Delta\kappa_Z + s^2/c^2 \Delta\kappa_\gamma, \qquad\qquad 9.$$

where s and c are shorthand for $\sin\theta_W$ and $\cos\theta_W$. This relation is, again, a consequence of restricting our attention to dimension-six operators; it is violated by dimension-eight operators (22).

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[4]The operator $\mathcal{O}_{DW}$ also makes other contributions to the triple gauge vertex.

The relations of Equations 4 and 9 are examples of how an effective field theory provides some guidance as to what new interactions to look for. Another example is the absence of anomalous couplings among neutral gauge bosons (γ, Z) when one's attention is restricted to dimension-six operators (22, 41).

The two relations among the anomalous triple gauge couplings show that the five couplings (λ_Z, λ_γ, Δg_1^Z, $\Delta \kappa_Z$, and $\Delta \kappa_\lambda$) are described by only three independent parameters. However, the BW basis contains only two operators that contribute directly to the triple gauge vertex, $\mathcal{O}_W$ and $\mathcal{O}_{\varphi WB}$ (42). This seems to imply that the anomalous triple gauge couplings are described by only two independent parameters in this basis, which apparently contradicts the claim that the physics is independent of the basis. In particular, it appears that $\Delta g_1^Z = 0$ in the BW basis.

The solution to this puzzle is that there are also indirect contributions to the anomalous triple gauge couplings. For example, in the BW basis the operator (not listed in **Table 1** because it includes fermions)

$$\mathcal{O}_{\varphi l}^{(3)} = (\varphi^\dagger i \overleftrightarrow{D}_\mu^I \varphi)(\bar{l} \tau^I \gamma^\mu l) \qquad \qquad 10.$$

affects the input parameter G_F taken from muon decay, causing a shift in $\sin^2 \theta_W$ that in turn contributes to Δg_1^Z and $\Delta \kappa_Z$. The relations of Equations 4 and 9 are still respected.

In the GGPR basis, six operators contribute directly to the anomalous triple gauge couplings: $\mathcal{O}_{3W}$, $\mathcal{O}_{HW}$, $\mathcal{O}_{HB}$, $\mathcal{O}_{2W}$, $\mathcal{O}_B$, and $\mathcal{O}_W$, which are the same as in the HISZ basis but with the HISZ operator $\mathcal{O}_{BW}$ replaced by the GGPR operators $\mathcal{O}_B$ and $\mathcal{O}_W$.[5] One obtains the two relations among the anomalous triple gauge couplings (Equations 4 and 9), demonstrating once again that the same physics is obtained in any basis (27, 28).

4. FLAVOR

Many dimension-six four-fermion operators contribute to processes in the Standard Model that are forbidden at tree level and suppressed at one loop via the GIM mechanism (43). These processes put constraints on the coefficients of these dimension-six operators that are of order $c_i/\Lambda^2 \sim (10^6 \text{ GeV})^{-2}$. If the scale of new physics is 10^6 GeV, there is little hope of observing even its indirect effects at the LHC. However, it is possible that the coefficients, c_i, are much less than unity, in which case Λ could be much lower than 10^6 GeV. Is there a natural explanation for why flavor-violating dimension-six operators might have small coefficients while flavor-conserving dimension-six operators have coefficients of order unity?

One such explanation goes under the name minimal flavor violation (44–46), which can be incorporated into the effective field theory approach (47, 48). The basic idea is that the coefficients of the dimension-six operators that mediate flavor-changing processes are suppressed by the same small factors that suppress these processes in the Standard Model.

For the remainder of this review, we invoke minimal flavor violation as a rationale for concentrating on flavor-conserving processes. Minimal flavor violation suggests that the largest flavor-violating effects are to be found in top quark physics.

5. BEYOND S AND T

A well-known parameterization of physics beyond the Standard Model is applicable to models with heavy particles ($\gg m_Z$) that couple only to the electroweak gauge bosons. These particles

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[5]The operator $\mathcal{O}_B$ contributes by modifying the mixing of the W^3 and B fields.

contribute to the gauge boson self-energies. The leading terms in an expansion of these self-energies are known as S, T, and U (49). A global fit to these parameters has been performed by the Gfitter group, and a plot of S versus T (assuming $U = 0$) may be found in Reference 50 (also see Reference 51).

The effective field theory approach to physics beyond the Standard Model may be viewed as a generalization of S and T to heavy particles that couple to more than just the electroweak gauge bosons. Using the conventions of Reference 52, the parameters $\hat{S}$ and $\hat{T}$ are represented by the coefficients of the HISZ operators

$$\mathcal{O}_{BW} = \Phi^{\dagger} \hat{B}_{\mu\nu} \hat{W}^{\mu\nu} \Phi, \tag{11.}$$

$$\mathcal{O}_{\Phi,1} = (D_{\mu}\Phi)^{\dagger} \Phi \Phi^{\dagger} (D^{\mu}\Phi), \tag{12.}$$

as

$$\hat{S} = -\frac{c}{s}\Pi'_{30}(0) = -c_{BW}\frac{m_W^2}{\Lambda^2}, \tag{13.}$$

$$\hat{T} = -\frac{\Pi_{33}(0) - \Pi_{11}(0)}{m_W^2} = -c_{\Phi,1}\frac{v^2}{2\Lambda^2}. \tag{14.}$$

The parameter $\hat{U}$ does not correspond to any dimension-six operator. It first arises via a dimension-eight operator.

Bounds on $\hat{S}$ and $\hat{T}$ yield bounds on c_{BW}/Λ^2 and $c_{\Phi,1}/\Lambda^2$ (**Figure 3**). However, once one opens up the parameter space to include all dimension-six operators, bounds on the coefficients of these two operators become considerably looser (28, 42, 53).
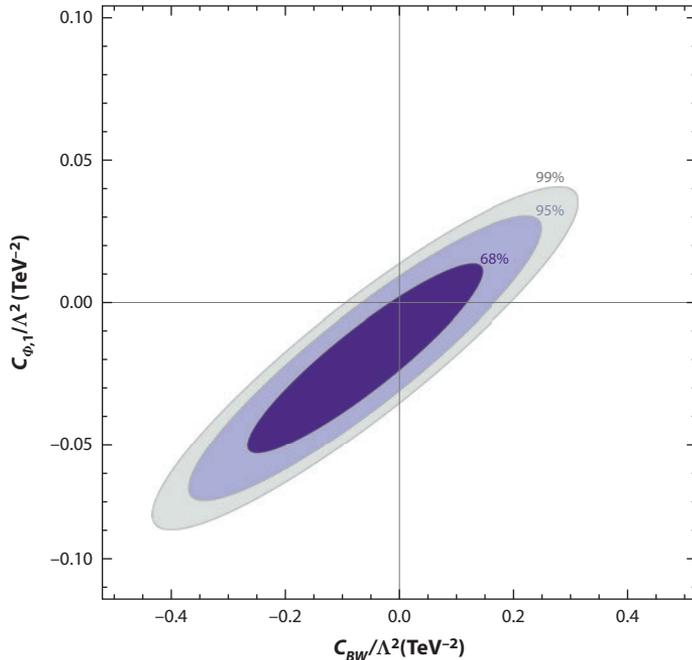

Figure 3

Bound on c_{BW}/Λ^2 and $c_{\Phi,1}/\Lambda^2$, assuming that all other operator coefficients vanish. The ellipses (*from outer to inner*) are at 99%, 95% and 68% CL.

To elucidate this point, consider the plot of c_{BW}/Λ^2 versus $c_{\Phi,1}/\Lambda^2$ in **Figure 3**. The strongest bound on c_{BW}/Λ^2 is obtained by setting $c_{\Phi,1}/\Lambda^2 = 0$; however, this is an ad hoc assumption. Letting $c_{\Phi,1}/\Lambda^2$ float loosens the bound on c_{BW}/Λ^2. The loosest bound on c_{BW}/Λ^2 is obtained by ignoring $c_{\Phi,1}/\Lambda^2$. However, doing so can be misleading, as it neglects the correlation between the bounds on c_{BW}/Λ^2 and $c_{\Phi,1}/\Lambda^2$.

The analogous effects appear when we add more operators. Letting the coefficients of these operators float loosens the bounds on both c_{BW}/Λ^2 and $c_{\Phi,1}/\Lambda^2$. However, there continues to be a correlation between c_{BW}/Λ^2 and $c_{\Phi,1}/\Lambda^2$, as well as a correlation between these coefficients and the coefficients of the additional operators.

In a different basis, S and T may be described by different operators. In the BW basis, they are described by the same operators as in the HISZ basis. In the GGPR basis, T is described by a similar operator as in the other two bases, but S is given by the sum of the coefficients of two operators that appear in neither the HISZ nor the BW basis:

$$\mathcal{O}_W = \frac{ig}{2}(H^\dagger \sigma^a \overleftrightarrow{D}^\mu H) D^\nu W^a_{\mu\nu}, \qquad 15.$$

$$\mathcal{O}_B = \frac{ig'}{2}(H^\dagger \overleftrightarrow{D}^\mu H) \partial^\nu B_{\mu\nu}, \qquad 16.$$

with[6]

$$\hat{S} = \frac{m_W^2}{\Lambda^2}(c_W + c_B). \qquad 17.$$

It may be misleading to exclude consideration of a dimension-six operator under the pretense that the coefficient of this operator is well constrained by precision electroweak data. A linear combination of such operators may be equivalent, via equations of motion or other identities, to an operator that is not well constrained (33). If one includes all the operators of a given basis in an analysis, this problem is avoided. Operators should be excluded only if it has been established that doing so will not bias an analysis.

6. PRECISION ELECTROWEAK MEASUREMENTS

Prior to the discovery of the Higgs boson, the most comprehensive analysis of precision electroweak measurements using effective field theory (54) was carried out in Reference 42. It included every dimension-six operator that affects precision electroweak experiments and imposed flavor symmetry to exclude flavor-changing operators; it also imposed *CP*. This analysis resulted in a list of 21 operators in the BW basis. A global analysis of the world's data was performed, resulting in a χ^2 distribution as a function of 21 coefficients. The bounds on these coefficients can be thought of as a 21-dimensional ellipsoid, the analog of the two-dimensional ellipse in the c_{BW}/Λ^2 versus $c_{\Phi,1}/\Lambda^2$ plane in **Figure 3**.

As discussed in Section 5, the bound obtained on the coefficient of an operator by arbitrarily setting all other operator coefficients to zero is too stringent. Conversely, the bound obtained on a coefficient by letting all other coefficients float can be misleading, as the correlation between coefficients is neglected. Both of these effects are exacerbated by the large dimensionality of the space of coefficients.

Two linear combinations of BW operators would not be bounded at all if one were to exclude the data from $e^+ e^- \to W^+ W^-$ (33). These are "blind directions" (55) with respect to the rest of

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[6]The operators in Reference 28 are suppressed by inverse powers of m_W instead of Λ, so the relation there is $\hat{S} = c_W + c_B$.

the precision electroweak data. The two linear combinations of BW operators are equivalent, via equations of motion, to the HISZ operators $\mathcal{O}_W$ and $\mathcal{O}_B$ or the GGPR operators $\mathcal{O}_{HW}$ and $\mathcal{O}_{HB}$. In this sense, the BW basis is not as transparent as the HISZ and GGPR bases in describing the precision electroweak data. These two directions, as well as the HISZ operator $\mathcal{O}_{WWW}$, are only weakly bounded by the $e^+e^- \rightarrow W^+W^-$ data, which have limited statistics.

We have found that the second-, third-, and seventh-smallest eigenvalues of the bilinear function χ^2 (42) have eigenvectors that span these three weakly bounded directions to a good approximation. The eigenvectors with the first-, fourth-, and fifth-smallest eigenvalues span a linear combination of quark–quark–lepton–lepton operators, and the eigenvector with the sixth-smallest eigenvalue corresponds to a combination of four-lepton operators.[7]

Because the directions corresponding to the HISZ operators $\mathcal{O}_W$ and $\mathcal{O}_B$ are present as linear combinations of operators in the BW basis, the correlations between operator coefficients are increased. This increase is reflected by the fact that the bound on the S parameter is loosened by a factor of $\sim$2,500 if all other operator coefficients are allowed to float versus setting them to zero (53). In the GGPR basis, in which these directions are included in the basis via the operators $\mathcal{O}_{HW}$ and $\mathcal{O}_{HB}$, this factor is reduced to $\sim$10 (28). A similar result is expected in the HISZ basis. Although any basis can be used to describe the data, the HISZ and GGPR bases have the advantage that the weakly bounded directions are included explicitly, which eliminates their correlation with the precision electroweak data.

With the discovery of the Higgs boson, more dimension-six operators can now be included. The analysis in Reference 42 sets the standard for future analyses.

7. CONSTRAINTS AT ONE LOOP

Some dimension-six operators are bounded only mildly. For example, the HISZ operators (also present in the BW basis; the second is also present in the GGPR basis)

$$\mathcal{O}_{WW} = \Phi^\dagger \hat{W}_{\mu\nu} \hat{W}^{\mu\nu} \Phi, \qquad \qquad 18.$$

$$\mathcal{O}_{BB} = \Phi^\dagger \hat{B}_{\mu\nu} \hat{B}^{\mu\nu} \Phi \qquad \qquad 19.$$

are bounded at tree level only by measurements of the Higgs boson coupling to electroweak bosons. Prior to the discovery of the Higgs boson, these operators were not bounded at all at tree level. However, these couplings also contribute to precision electroweak measurements at one loop. It is conceivable that the constraints on these operators from precision electroweak data are complementary to those from tree-level Higgs phenomenology (22, 29, 55–57).

Figure 4 shows the diagrams involving these operators that contribute to precision electroweak measurements. Because the diagrams are all self-energies containing heavy particles, we can use the S and T formalism to describe the result. An explicit calculation reveals that the contribution to $\hat{T}$ vanishes and the contribution to $\hat{S}$ is UV divergent. This means that the HISZ operator $\mathcal{O}_{BW}$ must be included in the analysis. Writing the final expression in the large m_H limit for simplicity, one finds (58–60)

$$\hat{S} = -\frac{c_{BW}(\mu)}{\Lambda^2} m_W^2 + \frac{g^2 m_W^2}{32\pi^2} \left(\frac{c_{WW}}{\Lambda^2} + \frac{s^2}{c^2} \frac{c_{BB}}{\Lambda^2} \right) \left[1 - 2\ln\left(\frac{m_H^2}{\mu^2} \right) \right], \qquad 20.$$

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[7] We thank W. Skiba for his assistance in this analysis.
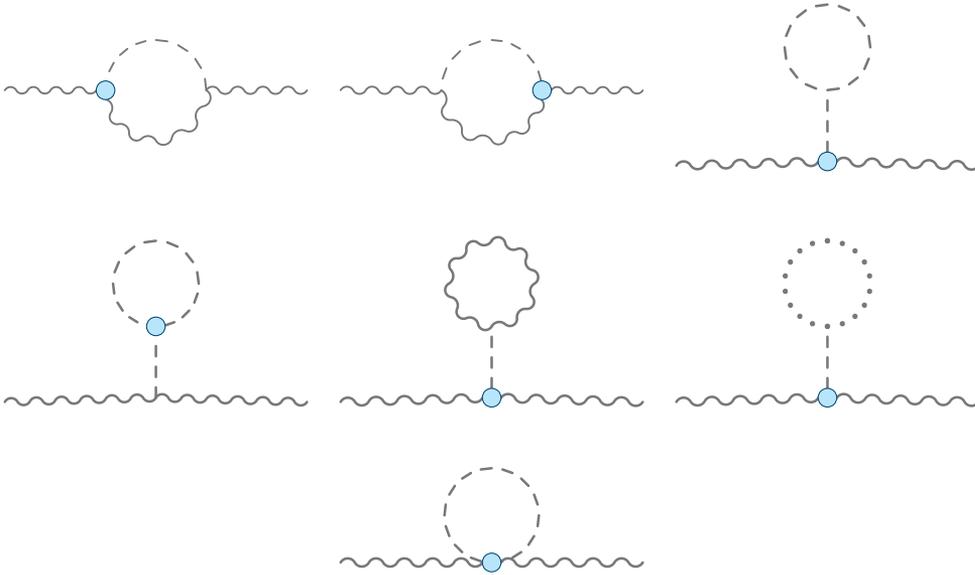
Feynman diagrams involving the operators $\mathcal{O}_{WW}$ and $\mathcal{O}_{BB}$, which contribute to precision electroweak measurements. The blue dots represent interactions arising from these operators.

where $c_{BW}(\mu)$ is the renormalized coefficient of $\mathcal{O}_{BW}$ at the renormalization scale μ in the $\overline{\text{MS}}$ scheme. Because this coefficient is unknown, it screens the contribution of the operators $\mathcal{O}_{WW}$ and $\mathcal{O}_{BB}$ to the $\hat{S}$ parameter. By contrast, the $\hat{U}$ parameter is UV finite, as it must be given so it does not correspond to any dimension-six operator:

$$\hat{U} = -\frac{g^2 s^2 m_W^2 m_Z^2}{8\pi^2 m_H^2} \frac{c_{WW}}{\Lambda^2}.$$ 21.

Performing a global fit of the precision electroweak data to c_{BW}, c_{WW}, and c_{BB} yields only very weak constraints on the last two coefficients (58–60). These constraints are much weaker than originally thought (22, 57). Those stronger bounds resulted from arbitrarily setting the renormalized coefficient $c_{BW}(\mu)$ to zero, combined with setting the renormalization scale to $\mu = \Lambda = 1$ TeV, which artificially enhances the one-loop contribution in Equation 20 (59). An analysis of the one-loop contribution of the HISZ operators $\mathcal{O}_{WWW}$, $\mathcal{O}_{W}$, and $\mathcal{O}_{B}$ to precision electroweak data yields similarly weak constraints (58–60).

8. HIGGS BOSON

The discovery of the Higgs boson has expanded the possibilities for probing dimension-six operators, leading to a flurry of activity in applying effective field theory to Higgs phenomenology (24, 25, 27, 28, 61–68). We anticipate that this subject will continue to evolve, so we can provide only a snapshot of the situation as of fall 2013. The upcoming run of the LHC at 13–14 TeV will probably fertilize further activity in this area.

The LHC data on Higgs production and decay are currently consistent with the predictions of the Standard Model, so we can merely put bounds on the coefficients of dimension-six operators involving the Higgs boson. Ideally, this should be done in the context of a global analysis so as

to include constraints on these operators coming from other measurements. Starting with the 21 operators in the BW basis needed to describe precision electroweak data identified in Reference 42, one needs to add several more operators to also describe Higgs boson production and decay. We count an additional 7 operators that are needed, bringing the total to 28. These are the BW operators $\mathcal{O}_{\varphi W}$, $\mathcal{O}_{\varphi B}$, and $\mathcal{O}_{\varphi\Box}$ from **Table 1**; the operator

$$\mathcal{O}_{\varphi G} = \varphi^\dagger \varphi G_{\mu\nu}^A G^{A\mu\nu} \qquad\qquad 22.$$

to parameterize the Higgs coupling to gluons; and three operators to parameterize the coupling of the Higgs boson to third-generation fermions,

$$\mathcal{O}_{u\phi} = \varphi^\dagger \varphi (\bar{q} u \tilde{\varphi}), \qquad\qquad 23.$$

$$\mathcal{O}_{d\phi} = \varphi^\dagger \varphi (\bar{q} d \varphi), \qquad\qquad 24.$$

$$\mathcal{O}_{e\phi} = \varphi^\dagger \varphi (\bar{l} e \varphi). \qquad\qquad 25.$$

At this time, there is no analysis that includes all 28 of these operators. Recent analyses (24, 25, 27, 28, 62–64, 66, 67) have been concerned mostly with describing the Higgs data, and include other data to varying degrees. One of the most extensive analyses includes 19 operators in the GGPR basis (28). Of the GGPR operators listed in **Table 1**, the operators $\mathcal{O}_{2W}$ and $\mathcal{O}_{2B}$ are eliminated in favor of other operators using equations of motion.[8]

An analysis of Higgs and W^+W^- data was performed (69) using six operators in the HISZ basis. This basis is incomplete (27) because two of the operators that were removed via equations of motion correspond to blind directions with respect to the precision electroweak data. These operators were reinstated in a revised version of Reference 69 that is available on the arXiv (70).

Let us consider the decay $H \to \gamma\gamma$, which has the most statistical significance. Because this decay proceeds at one loop in the Standard Model, dimension-six operators that contribute to this decay are strongly constrained. In the HISZ basis, the correction to the partial width for the decay receives direct contributions from dimension-six operators that are proportional to

$$\Delta\Gamma(H \to \gamma\gamma) \sim (c_{WW} + c_{BB} - c_{BW}). \qquad\qquad 26.$$

The same formula applies in the BW basis, with the operator coefficients appropriately renamed. In the GGPR basis,

$$\Delta\Gamma(H \to \gamma\gamma) \sim c_{BB}. \qquad\qquad 27.$$

In this basis it is evident that the bounds from $H \to \gamma\gamma$ are independent of bounds from electroweak vector boson pair production, which depend on the GGPR operators $\mathcal{O}_{3W}$, $\mathcal{O}_{HW}$, $\mathcal{O}_{HB}$, $\mathcal{O}_{2W}$, $\mathcal{O}_B$, and $\mathcal{O}_W$ (28).

After $H \to \gamma\gamma$, the decays with the most significance are $H \to Vf\bar{f}$, where $V = W$ or Z (53, 71). The dimension-six operators that contribute to these decays are not independent of those that contribute to electroweak vector boson pair production. At present the bounds on the operator coefficients from $H \to Vf\bar{f}$ and electroweak vector boson pair production are comparable in magnitude (28, 66).[9] This is less transparent in the BW basis, for reasons discussed in Section 6.

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[8]Reference 28 also lists the GGPR operator $\mathcal{O}_6 = \lambda|H|^6$ in its basis, but this operator cannot be probed at present.

[9]Reference 66 uses an incomplete basis, as discussed above for Reference 69. This problem has been remedied in a revised version that is available at arXiv.org (70).

The decay $H \rightarrow Z\gamma$, although it has not yet been observed, also bounds a linear combination of the same operator coefficients. In the GGPR basis (28),

$$\Delta\Gamma(H \rightarrow Z\gamma) \sim \frac{1}{4}(c_{HW} - c_{HB}) + 2s^2 c_{BB}, \qquad 28.$$

which depends on the coefficients of the operators $\mathcal{O}_{HW}$ and $\mathcal{O}_{HB}$ that affect electroweak vector boson pair production.

The situation becomes more interesting if deviations from the Standard Model are observed in the future. By measuring the coefficients of the dimension-six operators with good accuracy, we can hope to infer some or all features of the underlying theory. The coefficients of the dimension-six operators are measured at the electroweak scale, but the underlying theory resides at the scale Λ, so it is best to evolve the measured coefficients from the electroweak scale to the scale Λ to get the most accurate picture of the effective theory produced by the underlying physics. Fortunately, theorists are hard at work deriving the necessary machinery for this evolution (26, 27, 29, 68, 72–75). Unfortunately, as mentioned above, measuring the coefficients of the dimension-six operators does not reveal the scale Λ, only the ratio c_i/Λ^2. Thus, the coefficients must be evolved up to an arbitrary scale Λ, unless we are able to deduce the scale Λ by anticipating some or all of the details of the underlying theory. This is what happened with the electroweak theory, as recounted at the end of Section 1. It remains to be determined whether this evolution is important to unraveling the underlying physics.

To gain an understanding of the evolution of the operator coefficients, let us go back to Equation 20. Taking the derivative of this equation with respect to the renormalization scale μ gives

$$\mu\frac{d}{d\mu}c_{BW}(\mu) = \frac{g^2}{8\pi^2}\left(c_{WW} + \frac{s^2}{c^2}c_{BB}\right), \qquad 29.$$

which is the evolution equation for the coefficient $c_{BW}(\mu)$; in general, there are also contributions to the right-hand side from other operator coefficients, as well as Standard Model couplings. The coefficients of the operator coefficients on the right-hand side of this equation are referred to as the anomalous dimensions of c_{BW}, and the fact that the evolution depends on operator coefficients that are different from c_{BW} is referred to as operator mixing. Due to operator mixing, the anomalous dimensions of the 59 operators are best described by a matrix (27, 29, 73, 74). The anomalous dimension matrix was recently completed in the BW basis (68).

9. FINAL THOUGHTS

We hope the reader is convinced that the effective field theory approach is a simple and elegant way to treat new interactions beyond the Standard Model, with many virtues. It would be very exciting to discover new interactions and to use effective field theory to help unravel the underlying physics.

We would like to end with a dose of humility. Until a new collider is built, the energy frontier belongs to the LHC. Historically, hadron colliders, as well as proton fixed-target experiments, have discovered new physics by directly observing new particles, not by observing the effective interactions that these particles mediate at energies below their masses. Notable examples include the J/ψ, the Υ, the W and Z bosons, the top quark, and the Higgs particle. Although the W and Z bosons did manifest themselves first as four-fermion interactions, these interactions were observed in weak decays (W boson) and in neutrino neutral currents (Z boson), not in hadronic collisions. The observation of new physics via its low-energy effective interactions is unprecedented

in hadronic collisions. This fact can be viewed in a positive light; after all, we would rather discover the new particles directly than observe their low-energy effective interactions.

There are three facts behind this history. First, all of these particles are narrow resonances. Second, hadronic collisions probe a broad range of subprocess energies. Third, there are generally large backgrounds in hadronic collisions. A narrow resonance sticks up above the background, if there is enough subprocess energy to reach the resonance and enough parton luminosity to produce the resonance. The effective physics at energies below the resonance is not observable.

There is another historical point we would like to make. We have already recounted the story that led from the Fermi theory to the $V-A$ theory to the electroweak model and cited it as evidence of the usefulness of effective field theory. However, consider another effective field theory, that of the low-energy interactions of pions and kaons known as chiral perturbation theory (3). Although we now know that the underlying theory is QCD, chiral perturbation theory continues to be a useful tool. However, it was not chiral perturbation theory that led to the discovery of QCD. That discovery came via an entirely different route, which included the observation of scaling in deep-inelastic scattering and the theoretical realization of asymptotic freedom in non-Abelian gauge theories. Thus, the discovery and measurement of effective interactions, although important, may not be what allows us to deduce the underlying theory.

DISCLOSURE STATEMENT

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