ANNUAL REVIEWS

Relational Event Modeling

Federica Bianchi,[1] Edoardo Filippi-Mazzola,[1]
Alessandro Lomi,[2] and Ernst C. Wit[1]

[1]Institute of Computing, Faculty of Informatics, Università della Svizzera italiana, Lugano, Switzerland

[2]Faculty of Economics, Università della Svizzera italiana, Lugano, Switzerland;
email: alessandro.lomi@usi.ch

ANNUAL REVIEWS CONNECT

www.annualreviews.org

- Download figures
- Navigate cited references
- Keyword search
- Explore related articles
- Share via email or social media

Keywords

Abstract

Advances in information technology have increased the availability of time-stamped relational data, such as those produced by email exchanges or interaction through social media. Whereas the associated information flows could be aggregated into cross-sectional panels, the temporal ordering of the events frequently contains information that requires new models for the analysis of continuous-time interactions, subject to both endogenous and exogenous influences. The introduction of the relational event model (REM) has been a major development that has stimulated new questions and led to further methodological developments. In this review, we track the intellectual history of the REM, define its core properties, and discuss why and how it has been considered useful in empirical research. We describe how the demands of novel applications have stimulated methodological, computational, and inferential advancements.

1. INTRODUCTION

Statistical models for social and other networks are receiving increased attention, not only in specialized field journals such as *Network Science* or *Social Networks* but also in prominent interdisciplinary science journals such as *Science* (Borgatti et al. 2009, Butts 2009), *PNAS* (Stadtfeld et al. 2019), and *Scientific Reports* (Elmer & Stadtfeld 2020).

Attention to statistical models for networks is also on the rise in well-established generalist statistics journals, such as, for example, the *Journal of the American Statistical Association* (Hunter et al. 2008), the *Annals of Applied Statistics* (Snijders et al. 2010a), *Statistical Science* (Schweinberger et al. 2020), and the journals of the Royal Statistical Society (series A, B, and C) (Krivitsky & Handcock 2014, Gile & Handcock 2017, Vinciotti & Wit 2017, Koskinen & Snijders 2023). The *Annual Review of Statistics and Its Application* itself has recently demonstrated considerable interest in models for social networks by publishing comprehensive and up-to-date reviews on two popular classes of statistical models: exponential random graph models (ERGMs) (Amati et al. 2018) and stochastic actor-oriented models (SAOMs) (Snijders 2017).

Since the publication of these reviews, the availability of time-stamped data has increased due to innovations in data production, collection, storage, and retrieval technologies. It has been shown that network data samples based on observations collected at fixed time intervals are likely to miss fundamental differences in the timescales over which relational processes unfold (Golder et al. 2007). Computer-mediated communication (Lerner & Lomi 2023), sociometric badges (Wu et al. 2008, Stehlé et al. 2011, Elmer & Stadtfeld 2020), electronic trading platforms (Zappa & Vu 2021), online interaction logs (Tonellato et al. 2023), and video recordings (Pallotti et al. 2022) are just some of the new data-generating technologies capable of producing large quantities of relational event data connecting sender and receiver units.

During the same period, studies based on a variety of possible event-oriented designs have also become more common. While the empirical opportunities offered by relational event data have long been acknowledged by students of social networks (Freeman et al. 1987, Marsden 1990, Borgatti et al. 2009), statistical models affording a degree of temporal resolution consistent with the frequency of observed social interaction (Butts 2009) have become available only during the past fifteen years (Butts 2008).

Timescales vary considerably based on interaction settings. High-frequency transactions in financial markets (Bianchi & Lomi 2023) occur within seconds or fractions of seconds, while communication in emergency situations (Butts 2008, Renshaw et al. 2022) may take several minutes. Email exchanges (Perry & Wolfe 2013) can extend over hours, whereas interaction generated by more complex forms of social coordination among corporate actors may become observable only over days or even weeks (Amati et al. 2019). In these various cases, aggregating time-stamped relational event data into network ties defined over conventional, or convenient, time periods is unlikely to afford high-fidelity representations of the underlying interaction processes (Tuma & Hannan 1984).

Perhaps the main motivation that inspired the development of the relational event model (REM) proposed by Butts (2008) was to provide a general analytical framework where sequential ordering and timing replace concurrency and temporal aggregation of network edges as the "dominant concepts of phenomenal concern" (Butts 2008, p. 192) in the analysis of social interaction. This framework involves deriving, specifying, and estimating statistical models capable of assimilating and analyzing complex relational data without altering—through time aggregation—the natural time structure and sequential ordering of observed social interaction. Conversation (Gibson 2005), communication (Pilny et al. 2017), market exchange (Lomi & Bianchi 2021), and other, more complex forms of social coordination (Lerner & Lomi 2020a) can be understood

only with reference to the timing and sequential order of the relational events of interest (Abbott 1992), which contain important information that is typically lost when time-stamped events are aggregated into binary network ties (Pallotti et al. 2022).

Since its introduction, the REM has been significantly refined and adapted to an ever-increasing diversity and sophistication of emerging empirical problems (Butts et al. 2023). This review provides an opportunity to position relational event modeling in the broader context of statistical models for directed social interaction and assess the current state of field, incorporating a broad review of contemporary methodological, computational, and inferential developments.

The article is organized into eight main sections. Section 2 traces the intellectual origins and historical context of REMs. Section 3 identifies the observation plans and empirical research design elements typically associated with event-oriented studies of network dynamics. Section 4 surveys the available classes of REMs developed, at least in part, in response to specific problems with no satisfactory modeling solutions. Section 5 is dedicated to issues of empirical model specification and estimation. Section 6 examines the broad area of applications where REMs seem to have found fertile ground for development. Section 7 reviews the main challenges and open issues that orient current research. Section 8 concludes with a summary and a discussion of the main promises of relational event modeling.

2. HISTORICAL CONTEXT OF RELATIONAL EVENT MODELS

Contemporary statistical models for social and other kinds of networks rely heavily on the formalism of graph theory (Butts 2009)—an inheritance, in part, from earlier network models developed in sociometry (Moreno 1934, Jennings 1948), and within the structural tradition of social and cultural anthropology (White 1963, Lévi-Strauss 1971, Hage 1979, Barnes & Harary 1983, Hage & Harary 1984). Concepts that are central in contemporary statistical models for networks (Snijders 2001, Amati et al. 2018), such as those of degree, path distance, reciprocity, and transitive closure, are firmly rooted in the mathematical representation of networks as graphs (Barabási 2016).

The graph-theoretic formalism that inspired early Markov random graph models (Holland & Leinhardt 1977, 1981; Frank & Strauss 1986) may be considered the intellectual antecedent of contemporary statistical models for social networks (Wasserman & Pattison 1996, Snijders 2001, Snijders et al. 2006). Only recently have the limitations in representing networks of social relations as graphs started to become apparent. REMs entail an alternative understanding of social relations as emergent from sequences of relational events connecting a sender behavioral unit to one or more receivers (Butts 2008, 2009; Perry & Wolfe 2013; Lerner & Lomi 2023).

REMs provide a framework for analyzing and making inferences about the relational processes and dynamics in complex social systems. They are designed to capture the patterns of dependence in the occurrence and timing of relational events, such as communications, transactions, or social interactions, within a network.

Formally, REMs are rooted in event history models (for a review, see, e.g., Keiding 2014) expressing the hazards of an event to occur as a function of the history of previous events, as well as, potentially, additional nodal and relational attributes (Butts 2008). REMs afford analyses of how events unfold over time, how they are influenced by various factors, and how they, in turn, affect the structure and evolution of the network.

The exact hazards of a REM may follow different functional forms and specifications. In many instances, the hazard is assumed to be piecewise constant, resulting in wait times that are conditionally exponentially distributed. However, the precise definition of hazards will depend on the research questions and the empirical context, and these choices have implications for computational complexity, model fit, and interpretation of the model (Butts & Marcum 2017, Schaefer & Marcum 2017, Stadtfeld et al. 2017).

Two other modeling frameworks are appropriate for the analysis of longitudinal networks: the temporal ERGM and the SAOM. The formulation of the ERGM (Robins et al. 2007) may be understood in terms of the conditional probability of a network edge, given the relationships among all other network edges (Wasserman & Pattison 1996, Snijders et al. 2006). In the ERGM, the edges connecting pairs of senders and receivers are assumed to be interdependent, and such interdependencies can be captured by local configurations of network ties, such as triangles or star-shaped structures (Robins et al. 2007). More specifically, the existence of a tie in a social network is conditionally independent of ties that are far distant in space (Frank & Strauss 1986). Temporal extensions of the ERGM (Hanneke et al. 2010, Krivitsky & Handcock 2014) have been introduced to model network dynamics over time.

The SAOM (Snijders 1996, 2001, 2005, 2017) is a probability model evaluating change in network ties observed in adjacent time periods. While the SAOM takes into account relational states observed at multiple time points, it operates under the assumption that ties change continuously over time through a series of microsteps occurring between observations. Each microstep involves at most one alteration (Holland & Leinhardt 1977) in the network, with social actors choosing what ties to alter based on a multinomial choice probability model (McFadden 1973) accounting for the local configurations in which potential ties are embedded.

3. RELATIONAL DATA AND STUDY DESIGN

The relational event modeling framework was introduced to analyze the dynamics underlying social networks, with a focus on complex interactions and dependencies between behavioral units observed over time. REMs are flexible because they integrate temporal, structural, and attributional aspects of the data within a single framework. This flexibility sustains specification of very general yet detailed models of network evolution (Brandes et al. 2009).

3.1. Network Structure

The building block of the REM is the relational event, defined as an interaction initiated by a sender unit and directed toward one or more targets (Butts 2008). Examples include hospitals sharing patients (Vu et al. 2017, Amati et al. 2019), credit institutions exchanging financial assets (Lomi & Bianchi 2021, Zappa & Vu 2021, Bianchi & Lomi 2023), students telephoning each other (Stadtfeld & Block 2017), people interacting face-to-face (Elmer & Stadtfeld 2020) and in social groups (Hoffman et al. 2020), or employees exchanging emails (Quintane et al. 2013, Perry & Wolfe 2013).

Although relational events are typically defined between a single sender and a single receiver, generalizations are possible in which a sender may reach multiple receivers simultaneously. This situation is common in technology-mediated communication (Lerner & Lomi 2023), citation networks (Filippi-Mazzola & Wit 2023, Lerner & Hâncean 2023), and social gatherings attended by many participants (Lerner et al. 2021).

REMs can also model relations between units belonging to different classes in the context of more general bipartite processes. Examples include nonnative species invading new spatial niches (Juozaitienė et al. 2023), editors modifying Wikipedia pages (Lerner & Lomi 2017), computer developers fixing software problems (Tonellato et al. 2023), and political actors supporting or rejecting proposed legislation (Brandenberger 2018b, Haunss & Hollway 2022).

3.2. Sampling and Recording

Event network studies typically look at social units interacting within a bounded environment. Examples of geographical areas consist of student houses (Stadtfeld & Block 2017) or hospitals within

NOTATION

The REM describes temporal interaction between two (sets of) social units. We use the following notation:

- (t, s, r): a relational event at time t in which sender s interacts with receiver r
- $\lambda_{sr}(t)$: the rate/hazard at which sender s contacts receiver r at time t
- $\mathcal{H}_t$: filtration of the process, containing all information about relational events until time t
- L, L_P: likelihood and partial likelihood
- $\mathcal{R}(t)$: risk set at time t, consisting of all relational events that can happen at that time
- $\widetilde{\mathcal{R}}(t)$: sampled risk set at time t, consisting of the event (s, r) at time t and a set of sampled nonevents
- $x_{sr}(t)$: dyadic covariate(s) with corresponding effect(s) β
- $z_{sr}(t)$: dyadic covariate(s) with corresponding random effect(s) γ
- a: alter, i.e., an individual different from sender or receiver

geographical regions (Vu et al. 2017, Zachrison et al. 2022), whereas examples of situations with temporal boundaries involve interaction between emergency teams during different phases of a crisis (Butts 2008) or hospitals transferring patients on specific days of the week (Amati et al. 2019). Relational events taking place outside the main observation domain are usually not recorded, so the social networks reconstructed from streams of relational events are often incomplete. Unlike studies of statistically independent observations, network boundary specification might affect the validity, robustness, and replicability of the results (Laumann et al. 1989).

4. SPECIFICATIONS OF RELATIONAL EVENT MODELS

The units of analysis in the REM are the edges connecting individual pairs of senders and receivers. Those edges are typically stored in tuples (t, s, r), where s is the sender, r is the receiver, and t is the time of the relational event connecting s to r. At its core, the REM is defined as a point process for directed pairwise interactions that, in turn, are modeled through their rate function λ. The model assumes that λ may depend upon sender, receiver, past event history, and/or exogenous covariates. For more information on the notation used in this article, see the sidebar titled Notation.

4.1. Types of Relational Event Models

We consider a fixed time interval $[0,T]$, with $0 < T < \infty$, in which events occur. Events are defined as time-stamped interactions between senders and receivers. Both the sets of senders **S** and receivers **R** are assumed to be finite. For one-mode networks, the sets of senders and receivers overlap, $\mathbf{S} = \mathbf{R}$, whereas for two-mode or bipartite networks, they are distinct. The relational event process is a marked point process (Daley & Vere-Jones 2003, Cressie 2015) based on more general approaches (Borgan et al. 1995, Aalen et al. 2008) for event history sequences $\{(t_i, s_i, r_i) : i \geq 1, s_i \subset \mathbf{S}, r_i \subset \mathbf{R}\}$ and defined on a probability space $(\Omega, \mathcal{F}, P)$ adapted to the filtration $\mathcal{H}_t$, consisting of the history of process (Andersen et al. 1993). In principle, the marks (s, r) can be individuals or sets of senders and receivers.

Associated with this marked point process, we define a multivariate counting process N, whose components N_{sr} record the number of directed interactions between s and r,

$$N_{sr}(t) = \sum_{i \geq 1} \mathbb{1}_{\{t_i \leq t; \, s_i = s; \, r_i = r\}}.$$

According to the Doob–Meyer decomposition theorem (Meyer 1962), there exists a predictable process Λ_{sr} and a residual martingale process M_{sr}, such that the counting process can be

written as

$$N_{sr}(t) = \Lambda_{sr}(t) + M_{sr}(t).$$

The aim of the REM is to describe the structure of the predictable cumulative hazard process Λ_{sr}. By assuming that the counting process is an inhomogeneous Poisson process, we can write the cumulative hazard as

$$\Lambda_{sr}(t) = \int_0^t \lambda_{sr}(\tau)\,d\tau,$$

where λ_{sr} is the hazard function of the relational event (s, r). The general REM is defined as

$$\lambda_{sr}(t) = \mathbb{1}_{\{(s,r)\in\mathcal{R}(t)\}}\,\lambda_0(t)\,\psi\left\{\beta(t), x_{sr}(t)\right\}, \qquad \text{1.}$$

where $\lambda_{sr}(t)$ is only nonzero if the event (s, r) is contained in the risk set $\mathcal{R}(t)$ of possible events at time t, $\lambda_0(t)$ is the baseline hazard function unrelated to (s, r), ψ is a relative risk function as defined by Thomas (1981), $x_{sr}(t)$ is the $\mathcal{H}_t$ measurable set of endogenous and exogenous (possibly) time-varying variables, and $\beta(t)$ are the effect sizes. The function ψ is positive and, for identifiability, normalized by setting $\psi(\beta, 0) = 1$.

The relational event literature has focused on the exponential risk function $\psi(\beta, x) = \exp(\beta^\top x)$. Under the conditional independence framework (Besag 1974, 1975), network statistics are, by construction, dependent through space and time. Accordingly, the propensity of pairs of senders and receivers to mutually connect depends on which relational events they have sent or received in the past; their order in time, if not their exact timing; and exogenous factors like nodal or network attributes.

4.1.1. The original relational event model.
Butts (2008) introduced the REM as a piecewise homogeneous Poisson process, whereby the rate function is specified as

$$\lambda_{sr}(t) = \lambda_0\,\exp\left\{\beta^\top x_{sr}(t)\right\}\mathbb{1}_{\{(s,r)\in\mathcal{R}(t)\}}, \qquad \text{2.}$$

where $x_{sr}(t)$ is the collection of sufficient network statistics associated with the parameter vector $\beta \in \mathbb{R}^p$. The covariates in $x_{sr}(t)$ can depend on nodal characteristics, such as the age of r or the age difference between the pair (s, r), and also on the history of the process. For instance, the covariates $x_{sr}(t)$ can include a count of the number of past events between (s, r) before time t.

In the original formulation, the baseline rate of the interaction process λ_0 is assumed constant. This means that the waiting times between events are exponentially distributed, and once an event takes place, the context of the action is changed, and all the possible relational event waiting times are restarted.

4.1.2. Weighted and signed networks.
One of the first extensions of the original REM involved geopolitical conflicts and cooperation (Brandes et al. 2009). The authors highlighted that each interaction event between countries, international organizations, or ethnic groups could be given either a positive or negative weight according to the degree of cooperation or hostility between countries.

In this extended relational event definition (t, s, r, w), the interaction weight w is modeled alongside the event (t, s, r) as $p\{(t, s, r, w) \mid \mathcal{H}_{t-}\} = p\{(t, s, r) \mid \mathcal{H}_{t-}\} \times p\{w \mid (t, s, r), \mathcal{H}_{t-}\}$. The second term models the interaction weights depending on which countries are interacting and how they interacted in the past, whereas the first term is the usual REM likelihood, which is also allowed to depend on past interaction weights included in $\mathcal{H}_{t-}$.

Subsequent versions of the REM have accommodated weighted network statistics (Amati et al. 2019, Bianchi & Lomi 2023) associating a decay function $f(t, t_i, \alpha)$ to each past event (s_i, r_i), where t is the current time and α is a decay parameter. In its simplest formulation, the temporal relevance

of an event is assumed to decrease according to a power law, $f(t, t_i, \alpha) = (t - t_i)^{-\alpha}$, though other specifications may be used instead (Lerner & Lomi 2020b).

4.1.3. Multicast and polyadic interaction.

Multicast or polyadic interaction occurs when a single event links an individual sender to multiple receivers simultaneously. The possibility of one-to-many interaction was explicitly recognized by DuBois et al. (2013b) in their model of teacher-student interaction in a classroom context. In the same year, Perry & Wolfe (2013) provided a more general formulation specification of the intensity function for a sender $s \in \mathbf{S}$ addressing a set of receivers $R = \{r_1, \ldots, r_m\} \subset \mathbf{R}$, taking

$$\lambda_{sR}(t) = \lambda_{0s}(t; |R|) \, \exp\left\{\beta_0^\top \sum_{r \in \mathcal{R}_s(t)} x_{sr}(t)\right\} \prod_{r \in R} \mathbb{1}_{\{r \in R_s(t)\}}. \qquad 3.$$

The event rate function involves two types of stratification, discussed in greater detail in Sections 4.1.4 and 4.2.3, which explain the sender-specific definition of the risk set $\mathcal{R}_s(t)$ and baseline hazard, which also depend on the size of the receiver set. The network statistics are defined as the sums of the individual dyadic statistics $x_{sR}(t) = \sum_{r \in R} x_{sr}(t)$.

Lerner & Lomi (2023) introduced the relational hyperevent model (RHEM) by generalizing the definition of multicast network statistics. A hyperedge statistic x_{sR} is a function of the sender and the entire set of receivers that cannot, necessarily, be decomposed into the sum of dyadic covariates. An example of a hyperedge covariate is inertia, the tendency toward repeating past relational events, defined as

$$x_{sR}^{\text{inertia}}(t) = \sum_{t_i < t} f(t, t_i, \alpha) \mathbb{1}_{\{s_i = s, R_i = R\}}.$$

In email exchanges, for instance, inertia may result in the existence of a mailing list including a moderator communicating with exactly the same group of individuals. In contrast, unordered repetition captures interaction within a stable set of actors with turn-taking among the senders (Gibson 2005). Replacing dyadic covariates with their hyperedge counterparts not only provides a richer collection of network covariates but also improves model fit (Lerner & Lomi 2023).

Alternative approaches for modeling polyadic interactions have been proposed. Kim et al. (2018) introduced the hyperedge event model, assuming that dyadic intensities stochastically determine the sender of the next event and its receiver set. Mulder & Hoff (2021) introduced a latent variable model whereby all potential receivers are assigned to the receiver set on the basis of a suitability score that depends on the sender- and receiver-specific characteristics.

4.1.4. Separable intensity functions.

Stadtfeld et al. (2017) developed the dynamic network actor model (DyNAM), an actor-oriented model for relational event data built upon the same paradigm introduced by Snijders (1996, 2017). The distinctive feature of the DyNAM is that it explains social interaction in terms of "individuals' preferences, available interaction opportunities, and individuals' perception of the social network they are embedded in" (Stadtfeld & Block 2017, p. 318). Accordingly, the inferential framework of the DyNAM consists of modeling a composite process made of

1. the sender's decision to initiate a relational event at a certain point in time and
2. the sender's decision to address a specific receiver.

The DyNAM decomposes event rates into two sender-centered components, i.e.,

$$\lambda_{sr}(t) = \underbrace{\lambda_s(t)}_{\substack{\text{select sender } s}} \times \underbrace{p(r \mid s)}_{\substack{\text{sender } s \\ \text{addresses receiver } r}}. \qquad 4.$$

Similar to the paradigm of Snijders (2005), sender event rates λ_s are modeled through an exponential link function evaluating nodal attributes at individual and network levels,

$$\lambda_s(t) = \lambda_0 \exp(\theta^\top x_s) \mathbb{1}_{\{s \in \mathcal{R}(t)\}}.$$

5.

Following McFadden (1973), receiver choice is modeled via a multinomial distribution, i.e.,

$$p(r \mid s, \mathcal{H}_t) = \frac{\exp\{\beta^\top x_{sr}(t)\}}{\sum_{r' \in \mathcal{R}_s(t)} \exp\{\beta^\top x_{sr'}(t)\}}.$$

6.

The timing distribution does not explicitly depend on the receiver characteristics, and this typically sacrifices model fit in favor of an actor-oriented interpretation of the model parameters.

The DyNAM has been recently extended with the incorporation of the DyNAM-i (Hoffman et al. 2020), which explains sequences of joining and leaving events in the context of group-based interactions. This extension captures

1. the specific nature of group conversations and interactions that typically occur in cliques and
2. the need to align network modeling strategies with the increasing use of sensor technologies, such as Bluetooth or radio frequency identification badges, to detect collective interaction.

Vu et al. (2017) also exploited the separability of intensity functions by decomposing the stream of relational events into event times and event destinations. The separable sender intensity and receiver choice model adds more flexibility to the relational event framework, allowing for the separation between sender and receiver effects, and not only between event weights and dyads as in Brandes et al. (2009).

4.2. Network Covariates

Given the general expression of the REM hazard in Equation 1, the drivers of the relational process $x_{st}(t)$ describe known, $\mathcal{H}_{t-}$ measurable, quantities quantifying endogenous or exogenous statistics of the sender, the receiver, or both.
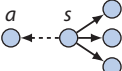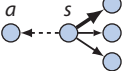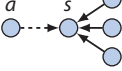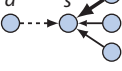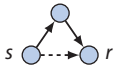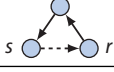
4.2.1. Endogenous versus exogenous covariates.
In statistical models for networks (e.g., Snijders et al. 2010b), covariates are endogenous to the extent that they depend on past interaction. Covariates are exogenous when they depend on characteristics of single nodes, i.e., monadic covariates, or pairs or nodes, i.e., dyadic covariates. One example of an endogenous covariate is reciprocity, while gender and geographical distance are exogenous covariates, representing monadic and dyadic characteristics, respectively. A basic selection of endogenous and exogenous covariates is displayed in **Table 1**. Other ad hoc specifications, such as those based on exchange sequences in conversational analysis, are explained by Butts (2008).

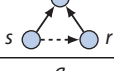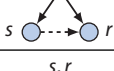An additional consideration refers to the hierarchy principle, whereby lower-order interaction terms should always be included in the presence of higher-order interaction terms (Pattison & Robins 2002, Wang et al. 2013). In the REM, for example, failing to account for heterogeneity of the senders and receivers may result in incorrect detection of triadic effects (Juozaitienė & Wit 2022a).

4.2.2. Heterogeneity.
There are two fundamental types of heterogeneity in event networks. Endogenous heterogeneity, or emergence, refers to the inherent stochasticity of the process itself combined with the dependence of future interactions on current ones. An example is virality, whereby, for instance, a paper gets cited because it was cited many times before. In the REM, endogenous heterogeneity can be captured by endogenous statistics.

Table 1 Basic menu of network statistics defined in the relational event model

Network statistic	Mechanism	Formula
Out-degree		$\sum\limits_{a\neq s} N_{sa}(t^-)$
Out-intensity		$\frac{1}{\text{out-degree}_s(t)}\sum\limits_{a\neq s}\sum\limits_{i:\, s_i=s, r_i=a} f\big(t, t_i, \alpha\big)$
In-degree		$\sum\limits_{a\neq s} N_{as}(t^-)$
In-intensity		$\frac{1}{\text{in-degree}_s(t)}\sum\limits_{a\neq s}\sum\limits_{i:\, s_i=a, r_i=s} f\big(t, t_i, \alpha\big)$
Repetition		$N_{sr}(t^-)$
Reciprocation		$N_{rs}(t^-)$
Transitive closure		$\sum\limits_{a\neq s,r} N_{sa}(t^-)N_{ra}(t^-)$
Cyclic closure		$\sum\limits_{a\neq s,r} N_{as}(t^-)N_{ra}(t^-)$
Sending balance		$\sum\limits_{a\neq s,r} N_{sa}(t^-)N_{ra}(t^-)$
Receiving balance		$\sum\limits_{a\neq s,r} N_{as}(t^-)N_{ar}(t^-)$
Node attribute		$x_{sr}(t)$
Node matching		$\text{dist}\{x_s(t) = x_r(t)\}$

Out-/in-degree/intensity statistics can refer to both senders and receivers. Node a is a third (alter) trading counterpart of the sender–receiver pair (s, r). The term $N_{sr}(t^-)$ is the number of relational events flowing from sender s to receiver r right before time t, while $f(t, t_i, \alpha) = (t - t_i)^{-\alpha}$ is the decay function accounting for the temporal relevance of previous relational events. In depicting network covariates, solid arrows ($\rightarrow$) refer to past relational events, while dashed arrows ($\dashrightarrow$) indicate current relational events. Thicker arrows indicate a higher intensity in the flow of relational events. Darker blue nodes indicate the presence of certain nodal exogenous statistics, while light blue nodes denote their absence.

The second type of heterogeneity, extrinsic variation, is either observed, such as the prestige of the institutions of the authors of a paper, or latent, such as the quality of the work the paper represents. Latent extrinsic heterogeneity in the REM can be expressed by means of random effects. Juozaitienė & Wit (2022a) and Uzaheta et al. (2023) proposed mixed effect extensions of the REM, i.e.,

$$\lambda_{sr}(t) = \mathbb{1}_{\{(s,r)\in\mathcal{R}(t)\}}\,\lambda_0(t)\,\exp\big\{\beta^\top x_{sr}(t) + \gamma^\top z_{sr}(t)\big\},$$

with dyadic covariates $z_{rs}(t)$ and $\gamma \sim N(0, \Sigma)$ the random effects. Estimation of the random effects variance can be done via expectation maximization (Dempster et al. 1977); Laplace approximations of the likelihood (Pinheiro & Bates 2006); or, in certain cases, a penalized zero-order spline approach (Wood 2017).

DuBois et al. (2013a) suggested a model that accounted for sender and receiver heterogeneity by means of a stochastic block structure. More recently, in an analysis of a communication network, Juozaitienė & Wit (2022a) showed that incorporating random effects for both the sender and receiver enhances the model fit compared with model specifications that solely rely on endogenous degree-based statistics. Therefore, the inherent differences between individuals in the network drive part of the heterogeneity in the interactions.

4.2.3. Stratification. Conceptually, stratification can be introduced either to model event streams in multiplex networks or to account for heterogeneity by specifying different baseline intensity functions for individual sets of dyads. Perry & Wolfe (2013) and Bianchi & Lomi (2023), for example, use sender-based stratification, effectively allowing each sender to have its own individual baseline hazard, i.e.,

$$\lambda_{sr}(t) = \lambda_{0s}(t) \exp\{\beta_0^\top x_{sr}(t)\} \mathbb{1}_{\{(s,r) \in \mathcal{R}(t)\}}.$$

Receiver-based stratification is defined in a similar fashion and usually occurs when there is heterogeneity in those nodes that are repeatedly targeted as receivers. In citation networks, for example, groundbreaking articles or patents have very distinct, individual citation profiles, which makes receiver-based baseline hazards an attractive option, given that they do not have to be estimated individually (Filippi-Mazzola & Wit 2023).

Juozaitienė & Wit (2022b) proposed a stratified version of the REM in which distinct baseline hazards are associated with distinct families of temporal network effects, such as reciprocity in its direct and generalized forms. Subsequent baseline hazard estimation reveals the tendency of some endogenous covariates to have very specific temporal effect profiles.

5. ESTIMATION AND COMPUTATION

The fundamental information about a sequence of relational events $\{(t_i, s_i, r_i) : i = 1, \ldots, n\}$ is contained in its likelihood function. For a REM, this function can be expressed as the product of the conditional generalized exponential event time densities and their associated multinomial relational event probabilities, i.e.,

$$L(\beta) = \prod_{i=1}^{n} \sum_{(s,r) \in \mathcal{R}(t_i)} \lambda_{sr}(t_i) \exp\left\{-\sum_{(s,r) \in \mathcal{R}(t_i)} \int_{t_{i-1}}^{t_i} \lambda_{sr}(\tau) d\tau\right\} \frac{\lambda_{s_i r_i}(t_i)}{\sum_{(s,r) \in \mathcal{R}(t_i)} \lambda_{sr}(t_i)}, \qquad 7.$$

where λ is a function of β and $\mathcal{R}(t)$ is the risk set, i.e., the set of all possible events that could have occurred at time t.

Estimating the parameters of REMs by maximizing the full likelihood poses several challenges. The likelihood function is indeed a complex object that involves explicit integration across the unknown hazard function and sums over large risk sets. In this section, we explore computational alternatives proposed to overcome the complexity of the full likelihood approach.

5.1. Partial Likelihood Estimation

The proportional hazards model (Cox & Oakes 1984) offers an attractive alternative to fully parametric models due to its absence of distributional assumptions regarding activity rates, which are then treated as nuisance parameters. It offers an effective simplification of the full REM likelihood
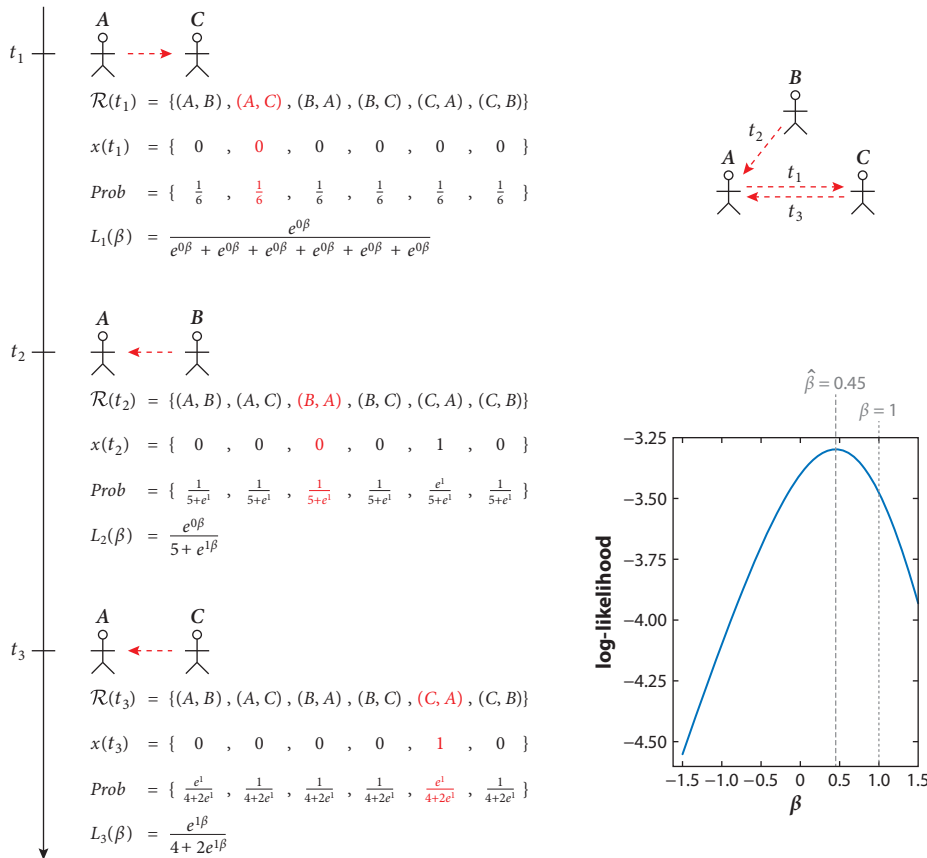
Figure 1

Example of a relational event process among three subjects $\{A, B, C\}$ based on the hazard function $\lambda_{sr}(t_i) = \lambda_0(t_i)\exp\{\beta x(t_i)\}$, with $\beta = 1$, whereby the event rate is influenced by the reciprocation statistic $x(t_i)$. The left side of the figure outlines the unfolding of the sequence of relational events according to the true probabilities *Prob*, mirrored in the graphical representation in the upper right. At each time point, the risk set $\mathcal{R}(t)$ consists of all six possible interactions between pairs of individuals. The red arrows correspond to the red relational event dyads that actually occurred. At time t_1, an interaction from A to C is observed, while all reciprocation covariates are zero, as no interaction has occurred yet. The probability of each event is determined using Equation 8, with partial likelihood $L_1(\beta)$. At time t_2, an interaction from B to A occurs. This event has partial likelihood $L_2(\beta)$, with one reciprocal nonevent in the denominator. At t_3, the reciprocal interaction from C to A is observed, with partial likelihood $L_3(\beta)$. The overall partial likelihood $L_P(\beta) = \prod_{i=1}^{3} L_i(\beta)$ is maximized at $\hat{\beta} = 0.45$, as shown on the lower right side of the figure, close to the actual effect size $\beta = 1$.

through the application of the partial likelihood L_P (Cox 1975) to counting processes (Andersen & Gill 1982) on network edges, which only involves multinomial event probabilities, i.e.,

$$L_P(\beta) = \prod_{i=1}^{n} \left(\frac{\exp\{\beta^\top x_{s_i r_i}(t_i)\}}{\sum_{(s,r)\in\mathcal{R}(t_i)} \exp\{\beta^\top x_{sr}(t_i)\}} \right). \qquad 8.$$

This expression eliminates the unknown baseline hazard, resulting in a more adaptive representation of the underlying network dynamics, while being able to estimate the parameters in a straightforward way by maximizing $L_P(\beta)$. An example is given in **Figure 1**.

As Butts (2008) noted, the partial likelihood corresponds to the full likelihood only when the event orderings are known, but not the exact timings. However, the partial likelihood approach faces a limitation in large networks, as the risk set in its denominator tends to expand quadratically with the number of nodes.

5.1.1. Risk set sampling.
The primary constraints in modeling the partial likelihood from Equation 8 are found in the risk set size. The idea of adopting subsampling strategies to approximate the partial likelihood was already noted by Butts (2008). Later, Vu et al. (2015) introduced a more efficient nested case-control sampling strategy (Borgan & Keogh 2015) to mitigate the computational complexity involved in estimating the partial likelihood.

Nested case-control sampling consists of sampling from the current risk set $\mathcal{R}(t)$, according to some probability $\pi\{\cdot \mid \mathcal{R}(t)\}$, a set of nonevents, or controls, for each event, or case. The sampled nonevents together with the events are called the sampled risk set $\widetilde{\mathcal{R}}(t)$. Borgan et al. (1995) showed that the sampled partial likelihood $\widetilde{L}_P$ accounting for the sampling probabilities is a valid likelihood. When this probability is assumed to be random, i.e., $\pi\{\cdot \mid \mathcal{R}(t)\} = 1/|\mathcal{R}(t) - 1|$, $\widetilde{L}_P(\beta)$ reduces to the simplified form, i.e.,

$$\widetilde{L}_P(\beta) = \prod_{i=1}^{n} \left(\frac{\pi((s_i, r_i)|\mathcal{R}(t_i)) \exp\{\beta^\top x_{s_i r_i}(t_i)\}}{\sum_{(s,r) \in \widetilde{\mathcal{R}}(t)} \pi((s,r) \mid \mathcal{R}(t_i)) \exp\{\beta^\top x_{sr}(t_i)\}} \right)$$

$$= \prod_{i=1}^{n} \left(\frac{\exp\{\beta^\top x_{s_i r_i}(t_i)\}}{\sum_{(s,r) \in \widetilde{\mathcal{R}}(t)} \exp\{\beta^\top x_{sr}(t_i)\}} \right), \qquad 9.$$

where $\widetilde{\mathcal{R}}(t)$ is the sampled risk set. Lerner & Lomi (2020b) employed nested case-control sampling to empirically showcase the efficiency of estimates on large networks, even when a limited number of nonevents is sampled.

5.1.2. Computational aspects of stratified relational event models.
Perry & Wolfe (2013) proposed the first REM that introduced sender stratification to expedite calculations and combined it with a customized method for maximizing the log partial likelihood. Vu et al. (2015) introduced a flexible stratification method allowing for data structures with many types of nodes and events, then showed the scalability of their approach to large data sets. Combining the two approaches, Bianchi & Lomi (2023) suggested a sender-stratified REM for high-frequency data, using nested case-control sampling to update the risk set at each new event. This model specification has been tested in empirical applications using millions of financial transactions. Filippi-Mazzola & Wit (2023) presented a receiver-stratified REM for the analysis of millions of patent citations, in which the hazard is modeled via smooth functions of the covariates using a spline approach.

5.2. Baseline Hazard Estimation

The prevailing method for the estimation of REMs is based on the partial likelihood method (Cox 1975), which treats the baseline hazard as a nuisance parameter. However, gaining insights into the temporal variations of the underlying event rates can be valuable for visualizing the baseline hazard.

Two common approaches to estimating the baseline hazard are the Breslow estimator (Breslow 1972) and the Nelson–Aalen estimator (Nelson 1972, Aalen 1978). Both estimate the cumulative hazard function only at the observed event times and so do not capture the continuous nature of the underlying baseline hazard function. Meijerink-Bosman et al. (2022a) estimated the baseline

hazard by assuming fixed baseline hazard rates within expanding windows. Juozaitienė & Wit (2022a) and Juozaitienė et al. (2023) improved smooth baseline hazard recovery by using a spline-based approximation.

5.3. Model Comparison and Diagnostic Tools

Traditional approaches, such as likelihood ratio tests, the Akaike information criterion, and the Bayesian information criterion, are widely adopted for comparing REMs (Foucault Welles et al. 2014, Pilny et al. 2016). Although model comparison methods are able to identify the best-fitting model in a set of competing models, they do not indicate whether the fit is adequate.

Butts & Marcum (2017) proposed an approach to model adequacy assessment based on deviance residuals and so-called surprise metrics. Similarly, Meijerink-Bosman et al. (2022a) suggested recall-based adequacy checking based on whether the observed events are in the top 5% of dyads with the highest predicted rates. Brandenberger (2019) introduced a procedure for model-based simulations of relational events. This method involves making predictions based on survival probabilities, which can then be used to generate new event sequences. In turn, comparing the simulated event sequences with the original data makes it possible to assess whether the model can accurately replicate network characteristics.

Measures borrowed from survival analysis can also be used to assess the goodness of fit of REMs. Juozaitienė et al. (2023) suggested using scaled Schoenfeld residuals to assess the proportional hazards assumption, deviance residuals to check for outliers and potential influential observations, and trends in martingale residuals to check for nonlinear effects of the covariates. Different model specifications are compared according to their deviance explained through pseudo R^2 measures (Cox & Snell 1989). Boschi et al. (2023) extended the martingale score process to evaluate the goodness of fit for fixed, smooth, and random effects.

Guidelines on the statistical accuracy and precision of the REM (summarized by Schecter & Quintane 2021) are defined by conducting experiments on simulated sequences of relational events, to which different sampling and scaling procedures have been applied. Meijerink-Bosman et al. (2022b) showed that the accuracy and precision of REM estimates depend on the width of the selected temporal window.

5.4. Bayesian Estimation of the Relational Event Model

REMs can be fitted also using Bayesian approaches (Butts 2008). DuBois et al. (2013b) extended the standard REM to incorporate multiple sequences, proposing a hierarchical model. Mulder & Leenders (2019) modeled multiple event sequences, estimating parameters that capture both within-sequence and between-sequence variations. This is particularly useful in multiplex networks, where multiple relational event sequences may be observed within the same network. Vieira et al. (2022) introduced a Bayesian hierarchical model that enables inference at the actor level, providing valuable insights into the drivers influencing actors' preferences in social interactions. Arena et al. (2022, 2023) proposed different solutions for studying memory decay in REMs, showing that the Bayesian approach allows the estimation of short- and long-term memory effects on the model parameters in relational event sequences.

5.5. Tools for Analyzing Relational Event Models

Limited specialized software tools are available for fitting REMs. The R-based packages `relevent` and `informR` (Marcum & Butts 2015) have been widely adopted and include all the REM features discussed by Butts (2008). Another R-based option is the `rem` package (Brandenberger 2018a), which allows the computation of endogenous statistics in signed one-, two-, and multi-mode

event networks. The `remstats` package (Meijerink-Bosman et al. 2022b) also assists empirical researchers in computing network covariates. It is typically used in combination with the `relevent` or `remstimate` package for model estimation. A further R-based option is `goldfish`, which mainly supports the DyNAM and is currently undergoing further development.

`eventnet` (Lerner & Lomi 2020b) offers a reliable and scalable Java-based interface for the computation of endogenous and exogenous statistics that serve as inputs of proportional hazards regressions. Bauer et al. (2021) and Fritz et al. (2021) showed that time-stamped relational data can be fitted through the well-established R package `mgcv` (Wood 2017) for generalized additive models with penalized likelihoods after a proper data reorganization.

One potential challenge for future studies is the development of a comprehensive package capable of computing network covariates under different sampling schemes while accommodating different model assumptions. Ideally, such a tool should be integrated within a suite of packages encompassing different estimation techniques and diagnostic tools as well.

6. APPLICATIONS OF RELATIONAL EVENT MODELS

Empirical studies adopting REMs span a wide range of disciplines. In this section, we classify these studies into established subjects, presented below in alphabetical order. Within each category, we offer an illustrative, rather than exhaustive, collection of research questions that have been investigated using REMs.

6.1. Communication

Within the field of communication studies, broadly construed, REMs have been adopted to analyze conversational processes within and between organizations and teams as well as in computer-mediated communication.

Butts (2008) and Renshaw et al. (2022) demonstrated the practical value of REMs in studies of organizational communication in the context of emergency management. REMs make it possible to specify covariates that capture basic conversational norms (Gibson 2003, 2005), such as the expectations of reciprocity in turn-taking.

Leenders et al. (2016) and Quintane & Carnabuci (2016) identified a number of challenges that hinder the identification of team dynamics and elaborate a REM-based analytic framework that supports a time-sensitive understanding of communication processes within teams. Similarly, Quintane et al. (2013), Pilny et al. (2016), and Schecter et al. (2018) revealed how REMs could be applied for studying the association between communication patterns and common indicators of process quality, coordination, and information sharing.

Computer-mediated communication is typically analyzed via two-mode REMs, which establish links between individuals and the situations they are involved in, such as the questions they answer in online question and answer communities (Stadtfeld & Geyer-Schulz 2011), or problems they attempt to resolve in open-source online projects (Quintane et al. 2014, Tonellato et al. 2023). In their study of communication instances in an online friendship network, Foucault Welles et al. (2014) adopted REMs to identify patterns of time dependence in data produced by online chats, focusing on how heterogeneous communication processes influence the creation, maintenance, and dissolution of communication ties over time.

6.2. Ecology

Studying behavior as sequences of relational events among animals promises to improve the understanding of key issues in behavioral ecology, such as how reciprocal giving and prosocial behavior emerge in small animal communities. Examples of this line of research include the study

of Tranmer et al. (2015) on group interactions among captive jackdaws, with a focus on their food sharing habits, and among cows struggling with the introduction of unfamiliar members in their community (Patison et al. 2015).

REMs have been adopted for studying ecological niche invasions (Juozaitienė et al. 2023) through the analysis of two-mode event networks linking invading species to territories. The relational event framework adopted in this study sheds light on potential risks associated with invasive species and provides insights into the ecological factors that may attract nonnative species.

6.3. Health and Healthcare

In health and healthcare research, REMs have been adopted in the study of social interaction in surgery rooms and interhospital patient transfers, with the aim of understanding how collaborations among healthcare units may or may not improve the quality of care. Pallotti et al. (2022) examined audio-visual recordings of task-related interactions among members of surgical teams to make sense of patterns of interpersonal communication among doctors and nurses organized around objects and technologies in the surgery room.

Lomi et al. (2014) studied interhospital mobility in a small community of hospitals and found that decentralized patient sharing decisions ensure patients' access to higher-quality healthcare services. Vu et al. (2017) showed that patient transfers are usually organized around small clusters of hospitals and include reciprocated patient exchange. Studying collaborations among hospitals in a regional community in southern Italy, Amati et al. (2019) explained that the generative mechanisms controlling change in event networks do not operate homogeneously and synchronously over time but instead vary systematically over different days of the week. Similarly, Zachrison et al. (2022) investigated the influence of characteristics such as reputation and institutional affiliation on the choice of destination hospital for emergency patients in the state of Massachusetts.

6.4. Political Science

Within the field of political science, REMs are frequently employed in their two-mode version. In the typical application, political actors are linked to social activities such as participation in cosponsoring events or support expressed for claims. Brandenberger (2018b) studied favor trading in congressional collaborations by examining the temporal dynamic of reciprocity, finding that the emergence of new collaboration clusters depends on the timing of mutual cosponsorship. Due to the variety of actors involved in the political debate, Haunss & Hollway (2022) adopted a multimodal extension of the DyNAM framework to study the political discourse around Germany's nuclear energy phase-out. This work identifies the potential discursive mechanisms that may have influenced the debate and when they may have operated.

REMs have occasionally been employed in criminology studies to investigate various aspects of offending behavior among individuals (Niezink & Campana 2022). These studies have explored phenomena such as illegal drug exchange in online markets (Duxbury & Haynie 2021, 2023) and the replication tendency of gang violence (Gravel et al. 2023). Overall, these analyses have demonstrated that criminal networks and personal attributes exert a substantial and temporally contingent influence on individual criminal acts.

6.5. Sociology

Because of the social science origin of REMs, sociology is perhaps the field where REMs have found the most extensive application. Restricting the focus to reciprocity, Kitts et al. (2017) and Lomi & Bianchi (2021) found that the mutual exchange of resources does not operate uniformly

across different exchange regimes, time frames, and material settings defined, for instance, by the value of resources being exchanged (Zappa & Vu 2021, Lomi & Bianchi 2021).

Lerner & Lomi (2017, 2020a) analyzed the emergence of status and hierarchies under conditions of extreme decentralization characterizing the Wikipedia crowdsourced project, in which independent volunteers interact by editing pages. In a study of individual editing behavior and collaboration/contention among Wikipedia editors, Lerner & Lomi (2019) examine the relation between team diversity, polarization, and productivity.

In the sociology of education, REMs have been applied to study how the nature of learning environments affects students' outcomes. Vu et al. (2015) studied educational experiences in massive open online courses by analyzing interactions between students with, and through, an online learning interface. DuBois et al. (2013b) investigated the social dynamics of high school classrooms by considering how the individual propensity to share information is affected by factors such as seating arrangements, teaching style, and sequences of participation shifts in conversation among students and teachers.

7. OPEN ISSUES AND CHALLENGES

In this section we describe a number of open challenges in the context of relational event modeling. Although not exhaustive, it points to a number of interesting directions in which we expect significant progress in the near future.

7.1. Procedures for Assessing Goodness of Fit

A pressing issue in relational event modeling involves the absence of a comprehensive procedure for assessing goodness of fit. Methods have been proposed involving recall adequacy and traditional residuals approaches. However, there exists no general consensus regarding a formal testing paradigm.

In general, what seems to be missing is an approach to testing goodness of fit of REMs consistent with established auxiliary variable approaches developed for ERGMs (Hunter et al. 2008) and SAOMs (Lospinoso & Snijders 2019).

7.2. Relational Big Data

Inference for REMs encounters a computational bottleneck as the number of relational events and, specifically, the number of actors increases. This presents practical implications, as it limits the applicability of REMs to large-scale networks. Addressing this limitation is an essential unresolved matter for REMs.

Building upon the risk set sampling concepts introduced by Vu et al. (2015) and Lerner & Lomi (2020b), Filippi-Mazzola & Wit (2023) proposed a stochastic gradient relational event additive model to analyze the network of patent citations in a dataset consisting of over 100 million events and 8 million nodes. By integrating case-control sampling with deep learning techniques, they have successfully achieved significant computational efficiency and real-time estimation of the model parameters.

7.3. Current Developments of Relational Event Models

Fritz et al. (2023) introduced a REM for spurious events (REMSE) to address the issue of events recorded through machine-coding errors, which can give rise to false positives and false negatives. The REMSE is presented as a robust tool suitable for studying relational events generated in potentially error-prone contexts. Its intensity is decomposed into two components: one associated

with true events and the other with spurious events. This decomposition allows for a more accurate representation and understanding of the underlying dynamics in the presence of potentially erroneous data.

Within the realm of coauthorship networks, Lerner & Hâncean (2023) adapted the RHEM (Lerner et al. 2021) to settings where events have a measurable outcome, such as a performance measure. These outcomes can serve as additional explanatory variables in the RHEM or can be used as response variables. This extension, known as the relational hyperevent outcome model, implies that event rates and relational outcomes are determined by the same explanatory variables utilized in the RHEM.

Incorporating relational event dynamics into a latent space or latent clustering allows for novel hypotheses about drivers of network formation. DuBois et al. (2013a) combined ideas from stochastic block modeling and REMs by partitioning the node set, where event dynamics within and between blocks evolve in distinct ways. Matias et al. (2018) developed a variational expectation-maximization method to estimate the latent groups. Moreover, combining REMs with latent space modeling allows the representation of actors as points in space, whose mutual distances drive the relational event process. Artico & Wit (2023) proposed a Kalman filter–based approach to estimate the trajectories of an actor in a latent space. An alternative approach is discussed by Rastelli & Corneli (2021), where the likelihood of an event given the current latent positions is maximized by stochastic gradient descent.

8. CONCLUDING REMARKS

Since their introduction fifteen years ago (Butts 2008), REMs have undergone considerable refinement (Brandes et al. 2009, Perry & Wolfe 2013), encouraged important extensions (Lerner & Lomi 2023), and enabled development of substantive applications (Vu et al. 2015, 2017, Lerner & Lomi 2020a). Progress on these various fronts contributed to establishing relational event modeling as one of the most promising frameworks for the analysis of dynamic network data.

REMs add to the existing set of statistical models for the analysis of dynamic networks the possibility of using the information contained in the sequential order of social interaction events when social interaction events used to be transformed into network ties defined at more aggregate timescales. Conversation (Gibson 2003), financial transactions (Bianchi & Lomi 2023), technology-mediated communication (Butts 2008), problem-solving (Tonellato et al. 2023), disaster management (Renshaw et al. 2022), and medical emergencies (Zachrison et al. 2022) are only a few examples of processes where the sequential timing of relational events is essential for understanding the underlying observation-generating mechanisms. In situations characterized by comparable sequential constraints on social interaction, statistical models that assume the concurrency of network ties leave unresolved problems related to the fact that network mechanisms operate over different time frameworks and are regulated by different time clocks (Bianchi et al. 2022).

This review outlined the core properties and the mathematical underpinnings of REMs by tracking the development of the original model since its appearance in 2008. We devoted special attention to the challenges posed by the estimation of REMs, and discussed the computational approaches proposed to address the complexities of the original model and its successive variants. We emphasized the flexibility of the relational event modeling framework, which allows empirical specification to account for endogenous as well as exogenous statistics that may affect observed patterns of interaction. We discussed the various ways in which time may influence the impact that past events may have on future events—an issue that may be considered an empirical feature of the data that should be accounted for, or an opportunity to develop theoretically inspired hypotheses about how time affects social interaction.

We intended our review to appeal to a broad audience comprising both empirically minded researchers confronting problems posed by the analysis of relational event data with complex temporal dependencies, and statisticians interested in the analytical opportunities offered by recent advances in dynamic stochastic models for social interaction. We expect that future progress in the modeling of relational events, and social networks more generally, will depend on the extent to which members of these communities will continue to discover areas of intersection for their interests.

DISCLOSURE STATEMENT

ACKNOWLEDGMENTS

LITERATURE CITED

Aalen O. 1978. Nonparametric inference for a family of counting processes. *Ann. Stat.* 6(4):701–26

Aalen O, Borgan Ø, Gjessing H. 2008. *Survival and Event History Analysis: a Process Point of View*. New York: Springer

Abbott A. 1992. From causes to events: notes on narrative positivism. *Sociol. Methods Res.* 20(4):428–55

Amati V, Lomi A, Mascia D. 2019. Some days are better than others: examining time-specific variation in the structuring of interorganizational relations. *Soc. Netw.* 57:18–33

Amati V, Lomi A, Mira A. 2018. Social network modeling. *Annu. Rev. Stat. Appl.* 5:343–69

Andersen PK, Borgan Ø, Gill R, Keiding N. 1993. *Statistical Models Based on Counting Processes*. New York: Springer–Verlag

Andersen PK, Gill RD. 1982. Cox's regression model for counting processes: a large sample study. *Ann. Stat.* 10(4):1100–20

Arena G, Mulder J, Leenders RTA. 2022. A Bayesian semi-parametric approach for modeling memory decay in dynamic social networks. *Netw. Sci.* **https://journals.sagepub.com/doi/full/10.1177/00491241221113875**

Arena G, Mulder J, Leenders RTA. 2023. How fast do we forget our past social interactions? Understanding memory retention with parametric decays in relational event models. *Netw. Sci.* 11(2):267–94

Artico I, Wit EC. 2023. Dynamic latent space relational event model. *J. R. Stat. Soc. Ser. A* 186(3):508–29

Barabási A. 2016. *Network Science*. Cambridge, UK: Cambridge Univ. Press

Barnes JA, Harary F. 1983. Graph theory in network analysis. *Soc. Netw.* 5(2):235–44

Bauer V, Harhoff D, Kauermann G. 2021. A smooth dynamic network model for patent collaboration data. *AStA Adv. Stat. Anal.* 106:97–116

Besag JE. 1974. Spatial interaction and the statistical analysis of lattice systems. *J. R. Stat. Soc. Ser. B* 36(2):192–225

Besag JE. 1975. Statistical analysis of non-lattice data. *J. R. Stat. Soc. Ser. D* 24(3):179–95

Bianchi F, Lomi A. 2023. From ties to events in the analysis of interorganizational exchange relations. *Organ. Res. Methods* 26(3):524–65

Bianchi F, Stivala A, Lomi A. 2022. Multiple clocks in network evolution. *Methodol. Innov.* 15(1):29–41

Borgan Ø, Goldstein L, Langholz B. 1995. Methods for the analysis of sampled cohort data in the Cox proportional hazards model. *Ann. Stat.* 23(5):1749–78

Borgan Ø, Keogh R. 2015. Nested case-control studies: Should one break the matching? *Lifetime Data Anal.* 21(4):517–41

Borgatti SP, Mehra A, Brass DJ, Labianca G. 2009. Network analysis in the social sciences. *Science* 323(5916):892–95

Boschi M, Juozaitienė R, Wit EJC. 2023. Smooth alien species invasion model with random and time-varying effects. arXiv:2304.00654 [stat.AP]

Brandenberger L. 2018a. `rem`: Relational event models. *R Package*, version 1.3.1

Brandenberger L. 2018b. Trading favors—examining the temporal dynamics of reciprocity in congressional collaborations using relational event models. *Soc. Netw.* 54:238–53

Brandenberger L. 2019. Predicting network events to assess goodness of fit of relational event models. *Political Anal.* 27(4):556–71

Brandes U, Lerner J, Snijders TAB. 2009. Networks evolving step by step: Statistical analysis of dyadic event data. In *2009 International Conference on Advances in Social Network Analysis and Mining*, pp. 200–5. New York: IEEE

Breslow NE. 1972. Discussion of Professor Cox's paper. *J. R. Stat. Soc. Ser. B* 34(2):216–17

Butts CT. 2008. A relational event framework for social action. *Sociol. Methodol.* 38(1):155–200

Butts CT. 2009. Revisiting the foundations of network analysis. *Science* 325(5939):414–16

Butts CT, Lomi A, Snijders TAB, Stadtfeld C. 2023. Relational event models in network science. *Netw. Sci.* 11(2):175–83

Butts CT, Marcum CS. 2017. A relational event approach to modeling behavioral dynamics. In *Group Processes: Data Driven Computational Approaches*, ed. A Pilny, MS Poole, pp. 51–92. New York: Springer

Cox DR. 1972. Regression models and life-tables. *J. R. Stat. Soc. Ser. B* 34(2):187–202

Cox DR. 1975. Partial likelihood. *Biometrika* 62(2):269–76

Cox DR, Oakes D. 1984. *Analysis of Survival Data*. London: Taylor & Francis

Cox DR, Snell EJ. 1989. *Analysis of Binary Data*. London: Taylor & Francis

Cressie NAC. 2015. *Statistics for Spatial Data*. New York: Wiley

Daley DJ, Vere-Jones D. 2003. *An Introduction to the Theory of Point Processes*. Vol. I: *Elementary Theory and Methods*. New York: Springer

Dempster AP, Laird NM, Rubin DB. 1977. Maximum likelihood from incomplete data via the EM algorithm. *J. R. Stat. Soc. Ser. B* 39(1):1–38

DuBois C, Butts C, Smyth P. 2013a. Stochastic blockmodeling of relational event dynamics. In *Proceedings of the 16th International Conference on Artificial Intelligence and Statistics (AISTATS)*, ed. CM Carvalho, P Ravikumar, pp. 238–46. Brookline, MA: Microtome

DuBois C, Butts CT, McFarland D, Smyth P. 2013b. Hierarchical models for relational event sequences. *J. Math. Psychol.* 57(6):297–309

Duxbury SW, Haynie DL. 2021. Shining a light on the shadows: endogenous trade structure and the growth of an online illegal market. *Am. J. Sociol.* 127(3):787–827

Duxbury SW, Haynie DL. 2023. Network embeddedness in illegal online markets: endogenous sources of prices and profit in anonymous criminal drug trade. *Socio-Econ. Rev.* 21(1):25–50

Elmer T, Stadtfeld C. 2020. Depressive symptoms are associated with social isolation in face-to-face interaction networks. *Sci. Rep.* 10:1444

Filippi-Mazzola E, Wit EC. 2023. A stochastic gradient relational event additive model for modelling US patent citations from 1976 until 2022. arXiv:2303.07961 [stat.CO]

Foucault Welles B, Vashevko A, Bennett N, Contractor N. 2014. Dynamic models of communication in an online friendship network. *Commun. Methods Measures* 8(4):223–43

Frank O, Strauss D. 1986. Markov graphs. *J. Am. Stat. Assoc.* 81(395):832–42

Freeman LC, Romney AK, Freeman SC. 1987. Cognitive structure and informant accuracy. *Am. Anthropol.* 89(2):310–25

Fritz C, Mehrl M, Thurner PW, Kauermann G. 2023. All that glitters is not gold: relational events models with spurious events. *Netw. Sci.* 11(2):184–204

Fritz C, Thurner PW, Kauermann G. 2021. Separable and semiparametric network-based counting processes applied to the international combat aircraft trades. *Netw. Sci.* 9(3):291–311

Gibson DR. 2003. Participation shifts: order and differentiation in group conversation. *Soc. Forces* 81(4):1335–80

Gibson DR. 2005. Taking turns and talking ties: networks and conversational interaction. *Am. J. Sociol.* 110(6):1561–97

Gile KJ, Handcock MS. 2017. Analysis of networks with missing data with application to the national longitudinal study of adolescent health. *J. R. Stat. Soc. Ser. C* 66(3):501–19

Golder SA, Wilkinson DM, Huberman BA. 2007. Rhythms of social interaction: messaging within a massive online network. In *Communities and Technologies 2007: Proceedings of the Third Communities and Technologies Conference, Michigan State University*, ed. C Steinfield, BT Pentland, M Ackerman, N Contractor, pp. 41–66. New York: Springer

Gravel J, Valasik M, Mulder J, Leenders RTAJ, Butts C, et al. 2023. Rivalries, reputation, retaliation, and repetition: testing plausible mechanisms for the contagion of violence between street gangs using relational event models. *Netw. Sci.* 11(2):324–50

Hage P. 1979. Graph theory as a structural model in cultural anthropology. *Annu. Rev. Anthropol.* 8(1):115–36

Hage P, Harary F. 1984. *Structural Models in Anthropology*. Cambridge, UK: Cambridge Univ. Press

Hanneke S, Fu W, Xing EP. 2010. Discrete temporal models of social networks. *Electron. J. Stat.* 5:585–605

Haunss S, Hollway J. 2022. Multimodal mechanisms of political discourse dynamics and the case of Germany's nuclear energy phase-out. *Netw. Sci.* 11(2):205–23

Hoffman M, Block P, Elmer T, Stadtfeld C. 2020. A model for the dynamics of face-to-face interactions in social groups. *Netw. Sci.* 8(S1):S4–25

Holland PW, Leinhardt S. 1977. A dynamic model for social networks. *J. Math. Sociol.* 5(1):5–20

Holland PW, Leinhardt S. 1981. An exponential family of probability distributions for directed graphs. *J. Am. Stat. Assoc.* 76(373):33–50

Hunter DR, Goodreau SM, Handcock MS. 2008. Goodness of fit of social network models. *J. Am. Stat. Assoc.* 103(481):248–58

Jennings HH. 1948. *Sociometry in Group Relations: A Work Guide for Teachers*. Washington, DC: Am. Counc. Educ.

Juozaitienė R, Seebens H, Latombe G, Essl F, Wit EC. 2023. Analysing ecological dynamics with relational event models: the case of biological invasions. arXiv:2303.06362 [stat.AP]

Juozaitienė R, Wit EC. 2022a. Nodal heterogeneity may induce ghost triadic effects in relational event models. arXiv:2203.16386 [stat.AP]

Juozaitienė R, Wit EC. 2022b. Non-parametric estimation of reciprocity and triadic effects in relational event networks. *Soc. Netw.* 68:296–305

Keiding N. 2014. Event history analysis. *Annu. Rev. Stat. Appl.* 1:333–60

Kim B, Schein A, Desmarais BA, Wallach H. 2018. The hyperedge event model. arXiv:1807.08225 [stat.ME]

Kitts JA, Lomi A, Mascia D, Pallotti F, Quintane E. 2017. Investigating the temporal dynamics of interorganizational exchange: patient transfers among Italian hospitals. *Am. J. Sociol.* 123(3):850–910

Koskinen J, Snijders TAB. 2023. Multilevel longitudinal analysis of social networks. *J. R. Stat. Soc. Ser. A* 186(3):376–400

Krivitsky PN, Handcock MS. 2014. A separable model for dynamic networks. *J. R. Stat. Soc. Ser. B* 76(1):29–46

Laumann EO, Marsden PV, Prensky D. 1989. The boundary specification problem in network analysis. In *Applied Network Analysis*, ed. R Burt, M Minor, pp. 18–34. Thousand Oaks, CA: SAGE

Leenders RTAJ, Contractor N, DeChurch LA. 2016. Once upon a time: understanding team processes as relational event networks. *Organ. Psychol. Rev.* 6(1):92–115

Lerner J, Hâncean MG. 2023. Micro-level network dynamics of scientific collaboration and impact: relational hyperevent models for the analysis of coauthor networks. *Netw. Sci.* 11(1):5–35

Lerner J, Lomi A. 2017. The third man: hierarchy formation in Wikipedia. *Appl. Netw. Sci.* 2(1):2–24

Lerner J, Lomi A. 2019. Team diversity, polarization, and productivity in online peer production. *Soc. Netw. Anal. Min.* 9:29

Lerner J, Lomi A. 2020a. The free encyclopedia that anyone can dispute: an analysis of the micro-structural dynamics of positive and negative relations in the production of contentious Wikipedia articles. *Soc. Netw.* 60:11–25

Lerner J, Lomi A. 2020b. Reliability of relational event model estimates under sampling: how to fit a relational event model to 360 million dyadic events. *Netw. Sci.* 8(1):97–135

Lerner J, Lomi A. 2023. Relational hyperevent models for polyadic interaction networks. *J. R. Stat. Soc. Ser. A* 186(3):577–600

Lerner J, Lomi A, Mowbray J, Rollings N, Tranmer M. 2021. Dynamic network analysis of contact diaries. *Soc. Netw.* 66:224–36

Lévi-Strauss C. 1971. *The Elementary Structures of Kinship*. Boston: Beacon

Lomi A, Bianchi F. 2021. A time to give and a time to receive: role switching and generalized exchange in a financial market. *Soc. Netw.* In press. **https://doi.org/10.1016/j.socnet.2021.11.005**

Lomi A, Mascia D, Vu DQ, Pallotti F, Conaldi G, Iwashyna TJ. 2014. Quality of care and interhospital collaboration: a study of patient transfers in Italy. *Med. Care* 52(5):407–14

Lospinoso J, Snijders TAB. 2019. Goodness of fit for stochastic actor-oriented models. *Methodol. Innov.* 12(3). **https://doi.org/10.1177/2059799119884282**

Marcum CS, Butts CT. 2015. Constructing and modifying sequence statistics for relevent using informR in R. *J. Stat. Softw.* 64(5):1–36

Marsden PV. 1990. Network data and measurement. *Annu. Rev. Sociol.* 16:435–63

Matias C, Rebafka T, Villers F. 2018. A semiparametric extension of the stochastic block model for longitudinal networks. *Biometrika* 105(3):665–80

McFadden D. 1973. Conditional logit analysis of qualitative choice behaviour. In *Frontiers in Econometrics*, ed. P Zarembka, pp. 105–42. New York: Academic

Meijerink-Bosman M, Back M, Geukes K, Leenders RTAJ, Mulder J. 2022a. Discovering trends of social interaction behavior over time: an introduction to relational event modeling. *Behav. Res. Methods* 55(3):997–1023

Meijerink-Bosman M, Leenders RTAJ, Mulder J. 2022b. Dynamic relational event modeling: testing, exploring, and applying. *PLOS ONE* 17(8):e0272309

Meyer PA. 1962. A decomposition theorem for supermartingales. *Ill. J. Math.* 6(2):193–205

Moreno JL. 1934. *Who Shall Survive?: A New Approach to the Problem of Human Interrelations*. Washington, DC: Nerv. Ment. Dis. Publ. Co.

Mulder J, Hoff PD. 2021. A latent variable model for relational events with multiple receivers. arXiv:2101.05135 [stat.ME]

Mulder J, Leenders RTAJ. 2019. Modeling the evolution of interaction behavior in social networks: a dynamic relational event approach for real-time analysis. *Chaos Solitons Fractals* 119:73–85

Nelson W. 1972. Theory and applications of hazard plotting for censored failure data. *Technometrics* 14(4):945–66

Niezink NM, Campana P. 2022. When things turn sour: a network event study of organized crime violence. *J. Quant. Criminol.* **https://doi.org/10.1007/s10940-022-09540-1**

Pallotti F, Weldon SM, Lomi A. 2022. Lost in translation: collecting and coding data on social relations from audio-visual recordings. *Soc. Netw.* 69:102–12

Patison KP, Quintane E, Swain DL, Robins G, Pattison P. 2015. Time is of the essence: an application of a relational event model for animal social networks. *Behav. Ecol. Sociobiol.* 69(5):841–55

Pattison P, Robins G. 2002. Neighborhood-based models for social networks. *Sociol. Methodol.* 32(1):301–37

Perry PO, Wolfe PJ. 2013. Point process modelling for directed interaction networks. *J. R. Stat. Soc. Ser. B* 75(5):821–49

Pilny A, Proulx JD, Dinh L, Bryan AL. 2017. An adapted structurational framework for the emergence of communication networks. *Commun. Stud.* 68(1):72–94

Pilny A, Schecter A, Poole MS, Contractor N. 2016. An illustration of the relational event model to analyze group interaction processes. *Group Dyn. Theory Res. Pract.* 20(3):181–95

Pinheiro J, Bates D. 2006. *Mixed-Effects Models in S and S-PLUS*. New York: Springer-Verlag

Quintane E, Carnabuci G. 2016. How do brokers broker? Tertius gaudens, tertius iungens, and the temporality of structural holes. *Organ. Sci.* 27(6):1343–60

Quintane E, Conaldi G, Tonellato M, Lomi A. 2014. Modeling relational events: a case study on an open source software project. *Organ. Res. Methods* 17(1):23–50

Quintane E, Pattison P, Robins G, Mol JM. 2013. Short- and long-term stability in organizational networks: temporal structures of project teams. *Soc. Netw.* 35(4):528–40

Rastelli R, Corneli M. 2021. Continuous latent position models for instantaneous interactions. arXiv:2103.17146 [stat.ME]

Renshaw SL, Livas SM, Petrescu-Prahova MG, Butts CT. 2022. Modeling complex interactions in a disrupted environment: relational events in the WTC response. *Netw. Sci.* 11(2):295–323

Robins G, Pattison P, Kalish Y, Lusher D. 2007. An introduction to exponential random graph ($p*$) models for social networks. *Soc. Netw.* 29(2):173–91

Schaefer DR, Marcum CS. 2017. Modeling network dynamics. In *The Oxford Handbook of Social Networks*, ed. R Light, J Moody, pp. 254–87. Oxford, UK: Oxford Univ. Press

Schecter A, Pilny A, Leung A, Poole MS, Contractor N. 2018. Step by step: capturing the dynamics of work team process through relational event sequences. *J. Organ. Behav.* 39(9):1163–81

Schecter A, Quintane E. 2021. The power, accuracy, and precision of the relational event model. *Organ. Res. Methods* 24(4):802–29

Schweinberger M, Krivitsky PN, Butts CT, Stewart JR. 2020. Exponential-family models of random graphs: inference in finite, super and infinite population scenarios. *Stat. Sci.* 34(4):627–62

Snijders TAB. 1996. Stochastic actor-oriented models for network change. *J. Math. Sociol.* 21(1-2):149–72

Snijders TAB. 2001. The statistical evaluation of social network dynamics. *Sociol. Methodol.* 31(1):361–95

Snijders TAB. 2005. Models for longitudinal network data. In *Models and Methods in Social Network Analysis*, Vol. 1, ed. PJ Carrington, J Scott, S Wasserman, pp. 215–47. Cambridge, UK: Cambridge Univ. Press

Snijders TAB. 2017. Stochastic actor-oriented models for network dynamics. *Annu. Rev. Stat. Appl.* 4:343–63

Snijders TAB, Koskinen J, Schweinberger M. 2010a. Maximum likelihood estimation for social network dynamics. *Ann. Appl. Stat.* 4(2):567–88

Snijders TAB, Pattison P, Robins G, Handcock MS. 2006. New specifications for exponential random graph models. *Sociol. Methodol.* 36(1):99–153

Snijders TAB, Van de Bunt GG, Steglich C. 2010b. Introduction to stochastic actor-based models for network dynamics. *Soc. Netw.* 32(1):44–60

Stadtfeld C, Block P. 2017. Interactions, actors, and time: dynamic network actor models for relational events. *Sociol. Sci.* 4:318–52

Stadtfeld C, Geyer-Schulz A. 2011. Analyzing event stream dynamics in two-mode networks: an exploratory analysis of private communication in a question and answer community. *Soc. Netw.* 33(4):258–72

Stadtfeld C, Hollway J, Block P. 2017. Dynamic network actor models: investigating coordination ties through time. *Sociol. Methodol.* 47(1). **https://doi.org/10.1177/0081175017709295**

Stadtfeld C, Vörös A, Elmer T, Boda Z, Raabe IJ. 2019. Integration in emerging social networks explains academic failure and success. *PNAS* 116(3):792–97

Stehlé J, Voirin N, Barrat A, Cattuto C, Isella L, et al. 2011. High-resolution measurements of face-to-face contact patterns in a primary school. *PLOS ONE* 6(8):e23176

Thomas DC. 1981. General relative-risk models for survival time and matched case-control analysis. *Biometrics* 37(4):673–86

Tonellato M, Tasselli S, Conaldi G, Lerner J, Lomi A. 2023. A microstructural approach to self-organizing: the emergence of attention networks. *Organ. Sci.* In press

Tranmer M, Marcum CS, Morton FB, Croft DP, de Kort SR. 2015. Using the relational event model (REM) to investigate the temporal dynamics of animal social networks. *Anim. Behav.* 101:99–105

Tuma NB, Hannan MT. 1984. *Social Dynamics Models and Methods*. San Diego, CA: Harcourt Brace Jovanovic

Uzaheta A, Amati V, Stadtfeld C. 2023. Random effects in dynamic network actor models. *Netw. Sci.* 11(2):249–66

Vieira F, Leenders RTAJ, McFarland D, Mulder J. 2022. Bayesian mixed-effect models for independent dynamic social network data. arXiv:2204.10676 [stat.ME]

Vinciotti V, Wit E. 2017. Preface to the themed issue on "Networks and Society." *J. R. Stat. Soc. Ser. C* 66(3):451–53

Vu DQ, Lomi A, Mascia D, Pallotti F. 2017. Relational event models for longitudinal network data with an application to interhospital patient transfers. *Stat. Med.* 36(14):2265–87

Vu DQ, Pattison P, Robins G. 2015. Relational event models for social learning in MOOCS. *Soc. Netw.* 43:121–35

Wang P, Robins G, Pattison P, Lazega E. 2013. Exponential random graph models for multilevel networks. *Soc. Netw.* 35(1):96–115

Wasserman S, Pattison P. 1996. Logit models and logistic regressions for social networks: I. An introduction to Markov graphs and *p**. *Psychometrika* 61(3):401–25

White HC. 1963. *An Anatomy of Kinship: Mathematical Models for Structures of Cumulated Roles*. Hoboken, NJ: Prentice-Hall

Wood SN. 2017. *Generalized Additive Models: An Introduction with R*. Boca Raton, FL: CRC Press. 2nd ed.

Wu L, Waber BN, Aral S, Brynjolfsson E, Pentland A. 2008. Mining face-to-face interaction networks using sociometric badges: predicting productivity in an IT configuration task. *SSRN.* **https://ssrn.com/abstract=1130251**

Zachrison KS, Amati V, Schwamm LH, Yan Z, Nielsen V, et al. 2022. Influence of hospital characteristics on hospital transfer destinations for patients with stroke. *Circ. Cardiovasc. Q. Outcomes* 15(5):e008269

Zappa P, Vu DQ. 2021. Markets as networks evolving step by step: relational event models for the interbank market. *Phys. A Stat. Mech. Appl.* 565(C):125557