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UNITY OF CONCEPTS IN THE STRUCTURE OF MATTER

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1 INTRODUCTION

In this age of increasing specialization it is comforting to realize that basic physical concepts apply to a wide range of seemingly diverse problems. Progress made in understanding one area may often be applied in many other fields. This is true not only for various fields of materials science but for the structure of matter in general. As examples we illustrate how concepts developed to understand magnetism, superfluid helium, and superconductivity have been extended and applied to such diverse fields as nuclear matter, weak and electromagnetic interactions, quark structure of the particles of high energy physics, and phases of liquid crystals.

Theoretical methods used in quantum field theory and in many-body problems of condensed-matter theory have much in common. For example, Greens function methods, Feynman diagrams, and renormalization group methods introduced for quantum electrodynamics have also been used in many problems in condensed-matter physics. The concept of spontaneously broken symmetry and associated phase transitions derived for condensed matter are now being widely used for problems of high energy physics. The commonality of physical concepts and methods extending over a broad range of problems is one of the main reasons that the National Science Foundation recently established an Institute for Theoretical Physics at the University of California, Santa Barbara. It is hoped that exchange of ideas between theorists in different disciplines will be mutually beneficial.

In this article we are not concerned with mathematical methods but with broad physical concepts useful for a wide variety of problems. One concept of great generality is the method of elementary excitations.

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Although it originated much earlier, the method in its modern form was developed in large part by Landau and co-workers. One tries to understand the ground-state and low-lying excitations that are approximate eigenstates of the system. The interactions between excitations generally are such that at low temperatures one may treat the system as a dilute gas of noninteracting excitations. At higher temperatures, interactions must be taken into account. In homogeneous systems, because of translational symmetry, excitations are waves and may be characterized by a wave vector, **k**. Examples are quasiparticle excitations and phonons in metals and spin waves in ferromagnets. When applied to particle physics, the ground state is the vacuum and the particles the excitations of the system.

When there is a phase transition, the ordered ground state and the low temperature phase may have lower symmetry than the Hamiltonian describing the system. Further, the ground state may be degenerate. A familiar example is the Heisenberg model of ferromagnetism in which at $T = 0\,\mathrm{K}$ the spins are aligned along any one of several preferred directions, or, in the isotropic case, along any direction in space. The low temperature phase may be characterized by an order parameter that in this case is taken to be the average magnetization, specified by its three components. Elementary excitations are spin waves in which the magnetization precesses about the direction in the ground state. With increasing temperature, the magnetization decreases and goes to zero at the Curie temperature, T_c, where there is a phase transition to the paramagnetic state.

It is not always easy to recognize the nature of the order that characterizes the broken symmetry of the low temperature phase. As first suggested on phenomenological grounds by Ginzburg & Landau, superconductors, as well as superfluid helium, are characterized by a complex order parameter with amplitude and phase. In the two-fluid model, the amplitude is proportional to the superfluid density and the gradient of phase to the superfluid velocity. At the nematic-smectic-A phase transition in liquid crystals, a density wave develops with oscillations along the axis parallel to which the molecules are aligned. A complex order parameter may be used to define the amplitude and phase of the wave.

Landau (1) suggested that properties in the vicinity of a second-order phase transition may be treated by expanding the free energy in powers of the order parameter. This implies a similarity of properties between different systems that are characterized by similar order parameters. Later we discuss analogies between superconductors and liquid crystals in the smectic-A phase.

In phases characterized by order parameters, there are excitations corresponding to space and time variations of the parameters. Small amplitude oscillations again may be described by a wave vector, **k**. If the forces are of short range, the frequency goes to zero with increasing wavelength, or as $\mathbf{k} \to 0$. Spin waves in ferromagnets are an example. The quanta of such oscillations are known in high energy physics as Goldstone bosons and correspond to particles of zero mass.

With long-range forces, such as Coulomb forces between electrons, the frequency may remain large as $\mathbf{k} \to 0$. An example is plasma oscillations from longitudinal density fluctuations of the electron gas in metals. In this case the frequency is very large, corresponding to the order of 10 eV. The quanta, plasmons, are not normally excited. In high energy physics, the corresponding particles are massive and are known as Higgs bosons. With Higgs bosons, spontaneously broken symmetry may be introduced through an order parameter without getting unwanted massless Goldstone bosons.

In addition to the small-amplitude oscillations, there may be large-amplitude nonlinear excitations that maintain their identity through collisions. They are now often called solitons, the term originating from the solitary water wave observed to flow down a canal following a sudden change in level. Examples are vacancies and interstitials as point defects in crystals, quantized vortex lines in superfluids as line defects, and Bloch walls between magnetic domains in ferromagnets as sheet defects. These play analogous roles in quite different systems. In three-dimensional crystals, point defects may exist in thermal equilibrium. In two-dimensional systems, dislocations and vortex lines become point defects. And in quasi-one-dimensional systems, walls between domains become point defects.

The Landau mean field theory does not apply to critical phenomena at temperatures very close to T_c. In this region fluctuations are large and important ones have wavelengths large compared with the lattice or interparticle spacing. It turns out that critical phenomena depend mainly on the number of space dimensions, d, and on the number of parameters, n, required to specify the order parameter. For example, in the Heisenberg ferromagnet, $d = 3$ and $n = 3$. For a superfluid or the smectic-A phase, $d = 3$ and $n = 2$. Fluctuations are greater in systems of lower dimensionality. In strictly one-dimensional systems they are so large that they prevent a phase transition above $T = 0$. We discuss later a number of closely related one- and two-dimensional systems.

In systems with many identical particles, the statistics of the particles (Einstein-Bose or Fermi-Dirac) plays an essential role in determining both the ground state and elementary excitations. The helium liquids composed of isotopes of mass three and mass four, and mixtures of the two, provide rich systems for studying the striking differences in behavior that follow from the statistics. One might expect that helium would form the simplest of liquids, but at very low temperatures it exhibits remarkably complex behavior. The Bose liquid, ^{4}He, becomes superfluid below the λ-transition (2.2 K), while ^{3}He is a normal Fermi liquid down to about 10^{-3} K where it undergoes a pairing transition analogous to that of electrons in a superconductor.

In the following we give some examples illustrating common features between magnetic systems, liquid helium, helium films, superconductors, and liquid crystals. We then indicate how concepts developed to understand these systems have been applied to the structure of nuclei, nuclear matter, and the particles of high energy physics. In these latter applications the concept of spontaneously broken symmetry plays a key role.

2 MAGNETIC SYSTEMS

The study of magnetic systems has yielded a great deal of information about phase transitions in general. The Heisenberg model has been studied for lattices in one, two, and three dimensions ($d = 1, 2, 3$) and for spin orientations in one, two, or three dimensions ($n = 1, 2, 3$) with all combinations possible (2, 3). In the Ising model ($n = 1$), only two spin orientations are possible. In analogous systems, up and down spin may be replaced by presence or absence of an atom (lattice gas model) or presence of an atom of type A or B (order-disorder systems in alloys). The case $n = 2$, for which the spin orientations are confined to a plane, gives systems analogous to superfluids with a complex order parameter.

As mentioned in the introduction, the Heisenberg model in three dimensions ($d = 3$) provides a good example of the method of elementary excitations. Depending on the sign of the interaction between neighboring spins, the ground state may be ferromagnetic or antiferromagnetic. The elementary excitations are spin waves, specified by a wave vector, $\mathbf{k}$, in which the spins precess about the direction in the ground state. In the ferromagnetic case the energy for small $\mathbf{k}$ is proportional to $\mathbf{k}^2$, as it would be for a free particle (momentum $\mathbf{p} = \hbar\mathbf{k}$).

The low temperature specific heat is proportional to $T^{3/2}$ as in a classical monatomic gas. The interaction between spin waves is small; it gives a leading term in the expansion of specific heat in powers of the temperature that goes as T^3.

In the paramagnetic phase above the Curie temperature, T_c, the spins become increasingly free to orient; as $T \rightarrow \infty$ the magnetic susceptibility approaches that of a system of free spins. For $T > T_c$, one may start from a system of free spins and treat the spin-spin interaction energy, J, by a perturbation expansion in powers of J/k_BT. Just above T_c, fluctuations are large and the expansion converges slowly, diverging as $T \to T_c$. The low temperature expansion in terms of spin waves is in half-integral powers of k_BT/J, or in inverse powers of the coupling constant.

Two-dimensional (2D) Heisenberg systems are of particular interest. In 1944 Onsager gave an exact solution of the 2D Ising model ($d = 2$, $n = 1$) that played a very important role not only for this and analogous systems but also for testing approximate methods required for more difficult problems.

An interesting system that has attracted a great deal of attention and for which the exact solution is not known is the 2DXY model, corresponding to $d = 2$, with classical spins, in which the spin-spin interactions include only the components in the plane of the lattice (3). There is a transition temperature, T_c, at which the susceptibility diverges and there is an essential singularity in the specific heat, but the anomaly in specific heat is so small that it is practically unobservable. Although there is local order, there appears to be no long-range order in the magnetization below T_c.

This model is analogous to several other models, including the 2D Coulomb plasma. Presumably a soliton-like excitation in which the spins tend to align along concentric circles surrounding a point defect plays an important role. A mathematically similar system with no long-range order in magnetization is the 1DXYZ model ($d = 1$, $n = 3$).

3 QUANTUM FLUIDS

In this section we briefly review the differences in the ground state and elementary excitations of liquid ^3He and ^4He that result from the difference in statistics (4). The gross structure of the liquids, as given, for example, by the pair distribution function, is very similar but the low temperature properties are strikingly different. The Bose liquid ^4He becomes superfluid in the phase He II at temperatures below the λ-transition at $T_\lambda = 2.2$ K, while ^3He remains a normal Fermi liquid down to temperatures of the order of 10^{-3} K.

The properties of He II can be accounted for in quantitative detail by a two-fluid model derived from a spectrum of elementary excitations proposed by Landau. Landau also first gave the correct description of normal Fermi liquids of interacting particles in terms of quasiparticle excitations and interactions between them. Liquid ^3He and ^3He–^4He mixtures have served as model systems for testing predictions of the Fermi liquid theory. First proposed on phenomenological grounds, both of Landau's theories have since been derived from microscopic theory.

The λ-transition is generally attributed to an Einstein-Bose condensation of helium atoms in the liquid. In the ground state of a noninteracting Bose gas, the particles are all in the ground state of zero momentum. At finite temperatures, particles are thermally excited to states of higher momenta, but up to the Einstein-Bose transition temperature (3.13 K for a gas of the density of ^{4}He), a finite fraction remain in the state $\mathbf{p} = 0$. If the ground-state wave function of the interacting system is expanded in terms of the momenta, the number, $n_{\mathbf{p}}$, in states of momentum $\mathbf{p} > 0$ gradually decreases with increasing $\mathbf{p}$ from a maximum at $\mathbf{p} = 0$ to near zero as $\mathbf{p} \to \infty$. However, a finite fraction, estimated to be about 10%, remain at the state $\mathbf{p} = 0$. Thus in the liquid at rest, the state $\mathbf{p} = 0$ is macroscopically occupied. If the liquid is flowing with a velocity $\mathbf{v}_s$, the state of macroscopic occupation is $\mathbf{p}_s = m\mathbf{v}_s$ rather than $\mathbf{p} = 0$. The fraction decreases from 10% at $T = 0$ to zero at T_λ, above which the liquid is normal.

In contrast, in the ground state of a Fermi system of noninteracting particles, each state of momentum $\mathbf{p}$ is doubly occupied by particles of opposite spin up to the Fermi momentum, $\mathbf{p}_F$, and those above are unoccupied. Thus $n_{\mathbf{p}} = 2$ for $\mathbf{p} < \mathbf{p}_F$ and $n_{\mathbf{p}} = 0$ for $\mathbf{p} > \mathbf{p}_F$. In the interacting system, $n_{\mathbf{p}}$ decreases from a maximum at $\mathbf{p} = 0$ to zero as $\mathbf{p} \to \infty$, but a discontinuity in occupancy remains at $\mathbf{p} = \mathbf{p}_F$. In a strongly interacting system such as ^{3}He, the discontinuity is small. In fact, for the same density a plot of the momentum distribution of the particles of ^{3}He is very much like that of ^{4}He, the differences being the discontinuity at $\mathbf{p}_F$ in ^{3}He and the finite fraction of particles in the state $\mathbf{p} = 0$ in ^{4}He.

The spectra of elementary excitations are quite different. In ^{3}He, there are quasiparticle excitations in one-to-one correspondence with those of the noninteracting system, specified by occupancy of a state of momentum $\mathbf{p}$ and spin σ above $\mathbf{p}_F$ or a missing particle or hole below $\mathbf{p}_F$. Excitations in ^{4}He may be specified by a momentum $\mathbf{p}$ or wave vector $\mathbf{k} = \mathbf{p}/\hbar$. As proposed by Landau, the energy is linear in k for small $\mathbf{k}$, goes over a maximum to the roton minimum at $\mathbf{k} = \mathbf{k}_r$ and increases again beyond. For small $\mathbf{k}$, the excitations correspond to the quanta of longitudinal sound waves and are the most important ones at very low temperatures. Rotons with $\mathbf{k}$-values near the minimum of the excitation spectrum at $\mathbf{k}_r$ are the most important for temperatures above about 1 K.

The existence of a state of macroscopic occupation $\mathbf{p}_s = m\mathbf{v}_s$ in the Bose liquid specifies a particular reference frame and breaks Galilean invariance in the same way that crystalline order does. In a normal liquid, the states available to the system are independent of the reference frame in which they are described, but in the superfluid the states depend on the state of macroscopic occupation. Of course $\mathbf{p}_s$ may take on different values in different reference frames, but it must be specified in order to specify the system.

The superfluid properties and the two-fluid model may be accounted for in terms of this picture. If the superfluid is flowing with velocity $\mathbf{v}_s$ in a narrow channel the excitations come into equilibrium with the walls. These thermal excitations decrease the flow from $\rho\mathbf{v}_s$ to $\rho_s\mathbf{v}_s$, where ρ is the total density and ρ_s is defined to be the superfluid density, equal to ρ at $T = 0$ and decreasing to zero at $T = T_\lambda$. If the walls are moving with velocity $\mathbf{v}_n$, the total flow is $\rho\mathbf{v} = \rho_n\mathbf{v}_n + \rho_s\mathbf{v}_s$, where $\rho_n = \rho - \rho_s$ is the normal component. This is the basis for the two-fluid model.

In quantum mechanics, Galilean invariance is related to the fact that the wave functions of a particle have an arbitrary phase factor. One may replace ψ by $\psi' = \psi \exp[i\chi(\mathbf{r})]$ if one replaces $\mathbf{p}$ by $\mathbf{p}' = \mathbf{p} - \hbar \operatorname{grad} \chi$. The local gauge group corresponding to this transformation is the unitary group U(1). Macroscopic occupation of the state $\mathbf{p} = \mathbf{p}_s$ gives a spontaneous breaking of this gauge group.

One may describe a $\mathbf{v}_s(\mathbf{r})$ that varies slowly in space by taking $\mathbf{p}_s(\mathbf{r}) = m\mathbf{v}_s(\mathbf{r}) = \hbar \operatorname{grad} \phi(\mathbf{r})$. Then except for a factor, $\phi(\mathbf{r})$ is the velocity potential for potential flow of the superfluid. This implies no vorticity in the flow, or curl $\mathbf{v}_s = 0$.

Vorticity may be introduced in the form of quantized vortex lines. On the axis of a line, the fluid is normal, $\rho_s = 0$. Superfluid may circulate around the axis, but the circulation is quantized to be an integral multiple of h/m. This follows from the requirement that $\phi(\mathbf{r})$ change by a multiple of 2π on a circuit of the axis so that the wave function is single valued. Vortex lines play an important role in the flow properties of superfluid helium. For a given total vorticity, the free energy is lowest if it is divided into an array of vortex lines of unit circulation, h/m.

Superfluidity is also observed in helium films only a few atoms thick. These may be regarded as essentially two dimensional. In two dimensions there is no reason to expect an Einstein-Bose condensation. Nevertheless, there must be an ordering that breaks gauge symmetry in such a way that the states available to the system depend on the superfluid velocity. Likewise, in the three-dimensional case the order may not be an exact Bose condensation into a momentum state but one to a related ordered state that also involves broken symmetry. In two dimensions as well as in three, the local superfluid velocity is given by the gradient of a phase.

4 METALS, SUPERCONDUCTIVITY

Various ground states are possible for electrons in metals. In normal metals, the ground state is the Fermi sea in which quasiparticle states below the Fermi surface are occupied by electrons of both spin directions and those above the surface are unoccupied. In the ferromagnetic ground state of the band model, the spins are aligned so that there is only one electron in each orbital in the expanded Fermi sea. In an antiferromagnetic alignment, orbitals for one spin may be different from those of the opposite spin, again requiring twice the number of orbitals required by normal metals. In both of these magnetic systems there are spin wave as well as quasiparticle excitations.

In another possible ground state, the electrons may form a lattice (Wigner lattice) and the excitations are vibrational modes of the lattice. If, as is quite possible, there is an antiferromagnetic spin ordering in the lattice, there will also be spin wave excitations. A Wigner lattice has not been confirmed in three dimensions, but it has been in systems of lower dimensions.

As emphasized by Overhauser (5), there may be macroscopic occupation of one or more spin density wave (SDW) states or charge density wave (CDW) states. In charge density waves, the ions oscillate and their charges are partially screened by associated motion of the conduction electrons. A CDW may be regarded as macroscopic occupation of a phonon state. A spin density wave exists in the ground state of chromium metal. Both SDWs and CDWs have been observed in layer structures (quasi-two-dimensional) (6) and in linear chain structures (quasi-one-dimensional (7).

In this section, we shall be concerned mainly with the superconducting phase in which pairing of electrons gives rise to a superfluid condensate with an energy gap for quasiparticle excitations at the Fermi surface. As early as 1950, Ginzburg & Landau (see 8) phenomenologically described the superconducting phase just below the transition temperature in terms of a complex order parameter given by an effective wave function, $\Psi(\mathbf{r})$, for the condensate. Both the superfluid density, ρ_s, and superfluid velocity, $\mathbf{v}_s$, of the two-fluid model are given by the function $\Psi(\mathbf{r}) = |\Psi(\mathbf{r})| \exp[i\phi(\mathbf{r})]$, with the density, ρ_s, of the condensate proportional to the square of the amplitude. A canonical momentum, $\mathbf{p}$, is given by the gradient of the phase, $\phi(\mathbf{r})$. If the electrons are coupled to a magnetic field through a vector potential, $\mathbf{A}$, the canonical momentum differs from the kinetic momentum, $m^*\mathbf{v}_s$. Ginzburg & Landau use the usual expression relating $\mathbf{p}$ and $\mathbf{v}_s$:

$$m^*\mathbf{v}_s = \mathbf{p} - (e^*/c)\mathbf{A},$$

where m^* and e^* are masses and charges of the basic units involved. It is now known from microscopic theory that because of pairing of electrons in the superconducting phase, $m^* = 2m$ and $e^* = 2e$.

The free energy density, $F(\mathbf{r})$, is obtained from $\Psi(\mathbf{r})$ by expanding in powers of $|\Psi(\mathbf{r})|^2$ and keeping terms to the fourth order. In addition there are kinetic energy terms proportional to $\mathbf{v}_s^2$, giving

$$F(\mathbf{r}) = \mathrm{Re}\,\{\Psi^*(\tfrac{1}{2}m^*)(\mathbf{p}-(e^*/c)\mathbf{A})^2\Psi\} + a|\Psi|^2 + \tfrac{1}{2}b|\Psi|^4.$$

A nonlinear Schrödinger-like equation determines the function $\Psi(\mathbf{r})$ that minimizes the total free energy. When there is no flow ($\mathbf{v}_s = 0$), a transition from the normal to superconducting phase occurs when $a(T)$ changes from positive to negative with decreasing temperature. Below T_c, the minimum $|\Psi|^2 = -a/b$ and, above T_c, $\Psi = 0$. Nonvanishing values of Ψ imply a broken symmetry in the superconducting phase.

The expression for F($\mathbf{r}$) is gauge invariant, corresponding to the possibility of the gauge transformation:

$$\mathbf{p} \rightarrow \mathbf{p}' + \hbar\,\mathrm{grad}\,\chi$$

$$\mathbf{A} \rightarrow \mathbf{A}' - (\hbar c/e^*)\,\mathrm{grad}\,\chi.$$

However, there is broken gauge symmetry because one must transform $\mathbf{p}$ along with $\mathbf{A}$. Thus $\mathbf{v}_s$ picks out a particular frame to which excitations of the systems must be referred, as in He II.

The Ginzburg-Landau equations apply to static rather than time-dependent problems. They apply only to the motion of the condensate, not the excitations of the system. And further, they do not include the effect of nonequilibrium distributions of excitations on the condensate. Time-dependent generalizations have been derived in which such effects can be included under appropriate conditions.

Although quantitatively correct only near T_c, the Ginzburg-Landau equations give a good qualitative description of many superconductive phenomena at all temperatures. Generalizations of the equations have been used for a wide variety of problems in condensed-matter and high energy physics: charge density waves, liquid crystals, the Weinberg-Salam theory of electrons and neutrinos, and many others.

Superconductors may be divided into two types, I and II, according to their behavior in a magnetic field (9). Those of type I exhibit a perfect Meissner effect; magnetic flux is excluded except for a small penetration depth, λ, near the surface. When the applied field exceeds the critical field, $\mathbf{H}_c$, a transition to the normal phase occurs. In type II superconductors, flux penetrates in the form of an array of quantized flux lines when the field is above a lower critical field, $\mathbf{H}_{c1} < \mathbf{H}_c$. The flux lines are analogous to the vortex lines in He II. Along the axis, $\Psi \rightarrow 0$. Currents of electrons circulating about the axis give a magnetic field parallel with the axis. A change of phase of 2π in $\Psi(\mathbf{r})$ in a circuit around the axis corresponds to a total of one unit of magnetic flux, hc/e^*.

According to the microscopic theory (BCS) (10), the wave function for the superconducting ground state may be regarded as a superposition of functions for low-lying normal configurations in which quasiparticle states of opposite spin and exactly the same total momentum are either both occupied or both empty. A wave function of this form takes advantage of an effective attractive interaction between electrons resulting from the electron-phonon interaction. The common momentum $\mathbf{p}_s = \mathbf{p}_1 + \mathbf{p}_2$ of the paired states $(\mathbf{p}_1, \sigma; \mathbf{p}_2, -\sigma)$ corresponds to the $\mathbf{p}_s$ of the Ginzburg-Landau theory. The momentum $\mathbf{p}_s(\mathbf{r})$ may vary slowly in space as described in that theory.

Quasiparticle excitations of the superconducting phase are in one-to-one correspondence with those of the normal phase but have an energy gap, 2Δ, for excitation of a pair of quasiparticles (excited particle and hole left behind) from the condensate. Not long after the first papers on the microscopic theory appeared Gor'kov showed that near T_c the Ginzburg-Landau equations do indeed follow from the microscopic theory with a complex $\Delta(\mathbf{r})$ playing the role of the order parameter, $\Psi(\mathbf{r})$.

When flux penetrates a type II superconductor, it is energetically favorable for it to divide into singly quantized flux lines. Normally, the lines start and end at the surface of the specimen, but they may also form closed loops in the interior. If magnetic monopoles existed, flux lines could presumably terminate on a monopole or carry flux from a positive to a negative monopole. The total flux carried by a flux line is that emanating from a monopole of magnetic charge $\mu^* = hc/4\pi e^*$. This is the Dirac unit for a basic electric charge e^*. For a superconductor, $e^* = 2e$, so that μ^* would be one half that for the basic charge e assumed by Dirac. While there is no evidence for magnetic monopoles, there are analogies in color gauge theories of quark matter, as we discuss later.

5 SYSTEMS OF LOW DIMENSIONALITY

There are many seemingly diverse one- and two-dimensional problems that are mathematically equivalent or are closely related. In two dimensions, quantized vortex lines with axes normal to the plane play the role of point defects. The energy of a single line is proportional to $\ln(L/a)$, where L is a cutoff distance dependent on the size of the system and a is the size of the core. Two vortices of opposite circulation attract one another with an energy varying as the logarithm of the distance between them, and the terms in the total energy proportional to $\ln(L/a)$ cancel. Similar laws apply in two dimensions to magnetic systems with point defects in which the spins orient in concentric circles surrounding the point, to edge and screw dislocations, and to point

Coulomb charges. The analogies between these 2D systems and the important role that the vortex-like defects have on their properties were pointed out by Kosterlitz & Thouless in 1973 (11). They introduced the concept of a long-range order called topological order, a concept that has had very broad implications in other areas.

The physics involved is perhaps most easily understood in terms of a theory of melting in a 2D solid, as suggested by Kosterlitz & Thouless (11) based on a dislocation model suggested by Nabarro. The long-range order in the solid is broken up by free dislocations present in thermal equilibrium. Below the melting point they are bound in pairs with Burgers vectors of opposite signs, but above they dissociate so that they can move independently. Thus they respond to an infinitesimal shearing stress.

Bound pairs may exist in thermal equilibrium below the melting point because each pair has a finite energy independent of the size of the system. In the expression for the free energy of free dislocations, $F = E - TS$, both the energy $W_0 \ln (L/a)$ and the entropy, $k_B \ln (L^2/a^2)$ per dislocation are proportional to $\ln (L/a)$. Thus there will be a critical temperature given by $2k_B T_c = W_0$ above which the contribution of the dislocations to the free energy becomes negative and the number of dislocations increases rapidly. This corresponds to the melting temperature. The theory of 2D melting has been greatly refined in the past few years, but the basic ideas of Kosterlitz & Thouless have been confirmed (12).

Analogous considerations apply to superfluid flow in helium films. The energy of an isolated vortex in a film of thickness d given by an integral of $\rho_s v_s^2 d/2$ in the region outside of the core of radius a is $W_0 \ln (L/a)$ with $W_0 = \pi \hbar^2 n \rho_s / m\rho$, where n is the number of atoms per unit area and m is the atomic mass. The surface density of superfluid atoms, $n\rho_s/\rho$, at the transition is then given by the universal relation, $n_s = n\rho_s/\rho = 2mk_B T_c/\pi\hbar^2$. This prediction has been confirmed by experiment (13). In this case the critical temperature, T_c, is that at which the order in the film breaks up and there is destruction of superfluid flow. The critical temperature is below that corresponding to T_λ and accounts for observations that ρ_s/ρ is finite, of the order of 0.5, when superfluid flow in the film is destroyed. At lower temperatures, persistent currents have been observed to flow for long periods in loops of thin He films in capillary tubes, indicating that there is indeed some sort of long-range order.

The critical temperature for the planar classical-spin model in two dimensions (the 2DXY model) is $k_B T_c = \pi J$, where J is the spin-spin coupling constant. The theory we have given applies only to the vortex excitations, which are of the soliton type. At low temperatures, spin wave excitations are the most important ones.

One can perhaps get a better physical understanding from the analogous Coulomb problem. Below T_c, there are bound pairs with charges of opposite sign interacting by the 2D logarithmic Coulomb potential. Above T_c, they break up into free charges that can move under the influence of a field. Thus the change is from an insulator into a conducting plasma of charges. To allow for a variable number of charges, one may treat systems in which there is a fixed chemical potential.

Note that to avoid large energies which increase with the size of the system, below T_c there must be equal numbers of positive and negative charges (or of vortices with opposite directions of circulation). Kosterlitz & Thouless (11) introduced the concept of topological order determined by the boundary values. In the Coulomb case the order is given by the net number of charges, $N = N_+ - N_-$, and there are corresponding numbers for dislocations or vortices. One cannot change N without introducing energies of the order of $W_0 \ln(L/a)$ that increase with the size of the system. It is the stability of the topological order that gives the long-range order required for elastic response of a 2D solid and for superfluid flow in helium films.

It is remarkable that these 2D problems are mathematically equivalent not only to a number of other 2D problems but also to several important problems in one dimension, including the 1DXYZ spin-$\frac{1}{2}$ Heisenberg chain, the time-dependent (1+1D) Sine-Gordon model used for 1D solitons, and the 1+1D Thirring model used as a model problem in high energy physics. Sólyom (14) has reviewed the relations between these closely related one- and two-dimensional problems.

6 LIQUID CRYSTALS

Generalizations of the Ginzburg-Landau theory have been used successfully to account for a number of phase transitions in liquid crystals. We shall discuss here just one example, the smectic-A phase mentioned in the introduction, which, as shown independently by de Gennes (15) and McMillan (16), bears a striking analogy to superconductivity. In both cases there is a complex order parameter; in superconductors it is the effective wave function, $\Psi(\mathbf{r})$, in the smectic-A phase it is the amplitude and phase of the density wave. A bending stress corresponds to an applied magnetic field.

When there is a bend, there is a change in direction of the directrix, $\mathbf{n}(x, y)$, a unit vector along which molecules are aligned and normal to the layers of high density. For small stresses, the bend is confined to a penetration region, depth λ, near the surface. As in superconductors, there is a coherence distance, ξ, and the ratio $\kappa = \lambda/\xi$ determines whether the liquid crystal behaves like a type I or a type II superconductor. In type II ($\kappa > 1$), it is favorable for dislocations to enter and thereby reduce the strain when the applied stress exceeds a critical value.

If the bend is in the XY plane, n_x and n_y change, but the spacing between the layers remains fixed. This requires that $\int \mathbf{n} \cdot d\mathbf{l}$ around a closed loop be a multiple of the spacing between layers, d. For a dislocation with a unit Burgers vector the integral is $\pm d$. This is analogous in the superconductor to a flux line of unit strength.

When the applied stress is small, the liquid crystal corresponds to a type I superconductor and stress and strain are confined to a penetration at the surface. For larger stresses corresponding to type II, dislocations enter the bulk and stress and strain are uniformly distributed through the bulk by means of an array of dislocation lines.

Liquid crystals have textures defined by patterns of the directrix lines. The boundary condition is that **n** be normal to the surface so that the surface is parallel to a layer of high density. One may have line and point singularities in the pattern of directrix lines called disclinations. A point singularity from which directrix lines radiate out to the surface is unstable; the singular point would move out to the surface. There are several types of line disclinations. We have discussed line singularities analogous to edge dislocations; screw disclinations are also possible and enter when there is a torque rather than a bending stress. Surface singularities form at a boundary region between two domains in which there is a difference in direction of the directrix lines.

7 SUPERFLUID HELIUM THREE

At very low temperatures ^3He undergoes a pairing transition to an ordered phase with a very complex structure (17, 17a). Two phases exist in the P-T diagram, ^3He-A and ^3He-B, which are well described by a pairing in the ^3P state rather than the ^1S state characteristic of superconducting metals. The order parameter has several components described by two unit vectors, **l** for orbital motion and **d** for the spin. In ^3He-A the energy gap is anisotropic and vanishes on the equator of a Fermi sphere oriented with the **l**-axis the polar axis. Only parallel spin pairing is included, so the gap parameter, $\Delta(\theta)$, has two components, Δ_{++} and Δ_{--}, corresponding to $m_s = 1$ and $m_s = -1$ respectively. In ^3He-B the third component, $m_s = 0$, corresponding to the spin combination $(+-)+(-+)$, is included and the energy gap is isotropic.

There is a weak magnetic dipolar interaction between nuclear spins which connects orbital and spin degrees of freedom. The lowest energy for ^{3}He-A is obtained when **l** and **d** are parallel. If as described in the preceding paragraph, the spin components refer to the z-direction, the **d**-vector lies in the $x-y$ plane. The lowest energy for ^{3}He-B is found for the **l**-vector rotated through an angle in the $x-y$ plane relative to the **d**-vector.

The fluids ^{3}He-A and ^{3}He-B have textures like a liquid crystal with properties dependent on the orientations of the vectors **l** and **d**. There are soliton modes, in which the **d**-vector rotates about the **l**-vector, that can be excited by applying a static magnetic field and then switching it off. The **l**-vector does not stay fixed but also oscillates in space. Such modes can be detected by observing damped oscillations in magnetization that occur after the applied field is reduced to zero.

8 NUCLEAR MATTER

The concept of pairing energy in nuclei existed for a long time, but it was only after the microscopic theory of superconductivity was derived that similar paired wave functions were applied to nuclei (18). It takes more energy to remove a neutron from the paired condensate than it does to remove an odd neutron that does not take part in the pairing. The odd particle goes into a state above the energy gap. Methods of superconductivity theory have been applied with great success to account for a wide variety of phenomena. The main difference is that one must take into account the finite and relatively small number of particles in a nucleus as well as the shell structure.

The centrifugal force of a rotating nucleus is equivalent in first order to the force of a magnetic field on a charged particle. The analogue of the Meissner effect is a decrease in moment of inertia to a value well below that corresponding to a rigid body. The outermost layers of the nucleus rotate while the inner part remains fixed.

The largest superfluid objects in the universe are neutron stars. There is strong evidence that the neutrons in the interior are in a paired superfluid state (19). It is thought that the pairing is in a ^{1}D state, although this is not certain. The superfluidity manifests itself by the long time required to spin down to a steady-state period, which follows a sudden change jump due presumably to a quake in the crust. We observe the rotation of the crust and it takes a long time for the crust to share its motion with the interior.

In the layer between the neutron core and the crust there are protons as well as neutrons, with the charge of the protons compensated by electrons. The protons presumably form a superconducting condensate through which the very large magnetic fields are carried by quantized flux lines as in a type II superconductor.

9 BROKEN SYMMETRY IN HIGH ENERGY PHYSICS

The two most successful theories of high energy physics are the Weinberg-Salam theory of leptons and the theory of mesons and baryons based on quantum chromodynamics (QCD). The latter theory is one of colored quarks held together by gluons. Gluons are massive bosons of non-Abelian gauge fields. One of the major problems is to understand confinement, why free quarks are not observed and why they appear only in color singlets. In both theories, the vacuum state is one of spontaneously broken symmetry.

Weinberg (20) (and independently Salam) constructed a model for leptons (electrons and neutrinos) based on interactions with gauge fields (with bosons as quanta) and a two-component field, ϕ, analogous to the order paramater field in superconductivity. In the initial Hamiltonian, electrons and neutrinos as well as the bosons of the gauge fields have zero mass. The leptons as well as the gauge bosons enter the theory symmetrically. Symmetry is broken by the field ϕ taking a vacuum expectation value. In a suitable gauge it may be written $\langle\phi_1\rangle = \lambda$, $\langle\phi_2\rangle = 0$. As a result, the electron acquires a mass while the neutrino remains massless. Of the gauge fields, the charged spin-1 bosons and one of the two neutral spin-1 bosons acquire mass. The second neutral boson remains massless and is identified with the photon.

The theory is modeled after a relativistic generalization of the Ginzburg-Landau theory introduced by Higgs (21). The field ϕ is the order parameter field and the bosons of the field are presumed to be massive (Higgs bosons). A number of predictions of the theory, including the existence of the neutral massive boson, have been confirmed by experiments done since the theory was proposed in 1967.

In the theory of quantum chromodynamics, quarks have quantum numbers designated by flavor and color. There is evidence for five flavors, and it is thought that there may be as many as six. There are three color quantum numbers, usually called red, green, and blue. Color singlets composed of equal amounts of the three colors are then called white. From experiments on electron-proton and electron-neutron scattering, quarks inside a nucleon behave as if they are essentially free (asymptotic freedom). Yet they are confined, implying that the energy increases with the distance to which they are separated. One picture is that a quark-antiquark pair is tied together by a string and that as the length of the string increases it is favorable for the string to break with a quark and an antiquark appearing at the two broken ends.

Considerable use has been made of analogy with superconductivity to try to understand the nature of the strings (22). Strong forces are carried by vector gauge fields analogous to the electric and magnetic fields of electrodynamics, with color charges taking the place of the electric and magnetic charges. In electrodynamics, there are no magnetic monopoles, but the analogue may exist in the non-Abelian gauge theories used for strong forces. In an influential paper, Nielson & Olesen (23) pointed out the analogy between the string model and flux lines in type II superconductors. If the strings carry the analogue of magnetic flux, color charges would be analogous to Dirac magnetic monopoles, as suggested by Nambu (24).

Mandelstam (25) suggested that if magnetic monopoles exist, circulating currents of magnetic charge could give rise to electric flux along the axis of a flux line, and the color charges would then be the analogue of electric rather than magnetic charges. The vacuum would correspond to a superconducting phase for magnetic charges that excludes electric fields.

Mesons consist of quark-antiquark pairs tied together by a string. Baryons consist of three quarks in a color singlet. In this case the strings from the quarks could terminate at a monopole singularity. Another possibility is that the presence of the quarks reduces the magnitude of the order of parameters so as to give a small region of normal phase in the superconducting vacuum. This would justify the MIT bag model in which the quarks are assumed to move freely in a small volume of space occupied by the meson or baryon.

Another direction in which there is much in common with condensed-matter physics is the use of lattice gauge theories, which put quarks on a lattice. This approach was introduced to problems of high energy physics by Wilson (26), and allows use of computational methods analogous to those used for spins on a lattice. One may use scaling theory to see how the parameters of the theory vary when for example, the energy per unit length of string is held fixed while the lattice constant is varied. Wilson has given an introductory review of scaling in an article in *Scientific American* (2). One of the main problems is to see how to go from one limiting region to another, from asymptotic freedom to strong coupling, to understand how quarks are confined.

There is currently considerable interest in a more general super-unification that combines weak, electromagnetic, and strong forces into the same symmetry group, again spontaneously broken by Higgs fields.

Considerable use is made in high energy physics of topological quantum numbers. It may be that conserved or nearly conserved quantities such as charge, lepton number, and baryon number depend on topological numbers. The basic particles then would be soliton singularities in the vacuum.

The structure of the world is exceedingly complex. Our present understanding is comparable to the knowledge that the Ginzburg-Landau theory gave of superconductivity when there was no understanding of the underlying microscopic theory. The problem is something like, although much more complex than, understanding the underlying structure of superfluid ^{3}He from measurements of the properties of the liquid made in the superfluid world. That the atoms that make up the liquid have a simple structure in spite of the very complex properties in the superfluid phase gives hope that the basic structure of the world we live in may also be relatively simple if it can be deciphered into its basic components.

10 CONCLUDING REMARKS

We have briefly reviewed some common themes that apply to a variety of physical phenomena in solids, quantum fluids, nuclear matter, and the structure of elementary particles. For example, in two-dimensional systems dislocation melting of solids, destruction of superflow in He II films, phase transitions in planar spin systems, and the transition from insulating to conductive behavior in a Coulomb plasma all have a common origin: the onset of the breaking of pairs of defects or charges of opposite sign. Line defects in three-dimensional systems, quantized vortex lines or flux lines, and dislocations account for similarities of behavior in superconductors, liquid crystals, and, it is hoped, color confinement of quarks.

Low temperature phases may be described by order parameters and have lower symmetry than the Hamiltonian describing the system, a familiar example being ferromagnetism. The ordered-state superfluids break Galilean invariance in such a way that one must specify the superfluid velocity in order to define the states accessible to the system. In superconductors the broken symmetry is that of gauge invariance. Similar breaking of symmetry gives massive gauge fields in particle physics.

It has been possible to give only a sampling of the vast literature of the many topics discussed briefly in this chapter. In only a few cases has the original literature been cited. We have referred to reviews and to recent references in which citations to earlier literature may be found.

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